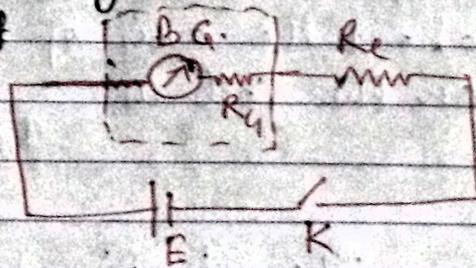


Differential equation of motion of moving coil of Moving coil galvanometer :

A. C. G. is connected in the circuit as shown in the circuit diagram.

Let R_g be the resistance of the coil galvanometer, R_e external resistance of the circuit.



Total resistance of the circuit, $R = R_e + R_g \rightarrow \textcircled{1}$

A cell of E.M.F 'E' is connected to the circuit. The maximum induced e.m.f in the galvanometer coil is $e = -N \frac{d\phi}{dt}$

$$= -N \frac{d}{dt} AB \cos \omega t$$

where $\phi = AB \cos \omega t$ - magnetic flux related with the coil. $= NAB \omega \sin \omega t$
 $= NAB \omega$

Maximum value of $\sin \omega t = 1$ $= NAB \frac{d\theta}{dt} \rightarrow \textcircled{2}$

where $\omega = \frac{d\theta}{dt}$ angular velocity of the coil.

\therefore Current flowing through the circuit

$$i = \frac{E - e}{R}$$

$$= \frac{E - NAB \frac{d\theta}{dt}}{R} \rightarrow \textcircled{3}$$

where θ - be the deflection of the coil in time t .

The deflecting torque on the coil is

$$\tau = NAB i$$

$$= NAB \left[\frac{E - NAB \frac{d\theta}{dt}}{R} \right]$$

$$= \frac{G_0}{R} \left[E - G_0 \frac{d\theta}{dt} \right] \rightarrow (4)$$

Where $G_0 = NAB$.

If I_m momentum inertia of the coil about the axis of rotation then torque on the coil is

$$I_m \frac{d^2\theta}{dt^2} \quad \left| \begin{array}{l} \therefore \tau = I_m \alpha \\ = I_m \frac{d^2\theta}{dt^2} \end{array} \right.$$

The retarding torque due to damping caused by air, viscosity, electromagnetic damping resulting from induced emf is proportional to angular velocity of the coil,

This retarding torque is $a_1 \frac{d\theta}{dt}$ where a_1 constant. The restoring torque on the suspension fibre is $c\theta$

$c \rightarrow$ restoring torque of suspension fibre due to unit twist.

In equilibrium of galvanometer

$$I_m \frac{d^2\theta}{dt^2} + a_1 \frac{d\theta}{dt} + c\theta = \frac{G_0}{R} \left[E - G_0 \frac{d\theta}{dt} \right]$$

$$\Rightarrow I_m \frac{d^2\theta}{dt^2} + \left[a_1 + \frac{G_0^2}{R} \right] \frac{d\theta}{dt} + c\theta = \frac{G_0 E}{R}$$

$$\Rightarrow I_m \frac{d^2\theta}{dt^2} + b_1 \frac{d\theta}{dt} + c\theta = \frac{G_0 E}{R} \quad \left| \begin{array}{l} \text{where} \\ b_1 = \left(a_1 + \frac{G_0^2}{R} \right) \end{array} \right.$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{b_1}{I_m} \frac{d\theta}{dt} + \frac{c}{I_m} \theta = \frac{G_0 E}{I_m R}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + 2b \frac{d\theta}{dt} + K\theta = f \rightarrow (5)$$

Equation (5) is the differential eqn of motion of coil of B.G.

(In the next class solution of this eqn.)

$$\text{where } b = \frac{b_1}{2I_m}$$

$$K = \frac{c}{I_m}$$

$$f = \frac{G_0 E}{I_m R}$$