

Ch: Atmospheric pressure.

1. Boyle's Law: The law states that if the temperature remains constant, the pressure varies inversely as the volume v .

i.e. $p \propto \frac{1}{v}$ or $p v = \text{constant}$

If m be the given mass of gas and ρ its density, then $m = \rho v$.

$$\therefore p \cdot \frac{m}{\rho} = \text{constant}$$

$\Rightarrow p = k\rho$, where k is a constant

2. Charles' Law: The law states that if the pressure remains constant, the volume increases by a definite fraction of the volume at 0°C for every degree Celsius through which the temperature increases.

Then if v_0 and v be the volumes of the gas at 0°C and $t^\circ\text{C}$, we have

$$v = v_0(1+\alpha t),$$

where $\alpha = \frac{1}{273}$ is called the coefficient of expansion.

Again mass $= p v = \text{constant}$,

i.e. $p v = p_0 v_0$

$$\text{or } \frac{p}{p_0} = \frac{v_0}{v} = \frac{v_0}{v_0(1+\alpha t)} = \frac{1}{1+\alpha t}$$

$$\Rightarrow p_0 = p(1+\alpha t)$$

Also, when temperature is constant, we have from Boyle's law at 0°C ,

$$\text{if } p = K p_0 = K(1+\alpha t)$$

Here K is a constant depending on the gas.

Height of a station by barometer:

case I. When temperature is supposed to be uniform;

Let p be the pressure, ρ the density of the air at a height z above the earth's surface, then

$$dp = -g\rho dz \quad \text{--- (1)}$$

$$\text{Also by Boyle's Law, } \rho = p/K \quad \text{--- (2)}$$

$$\text{Dividing (1) by (2), } \frac{dp}{p} = -\frac{g}{K} dz$$

$$\text{Integrating, } \log p = -\frac{g}{K} z + C \quad \text{--- (3)}$$

If p_0, ρ_0 denote the pressure and density at a height z_0 , then

$$\log p_0 = -\frac{g}{K} z_0 + C \quad \text{--- (4)}$$

Subtracting (4) from (3),

$$\log \frac{p}{p_0} = -\frac{g}{K} (z - z_0) \quad \text{--- (5)}$$

Now, if p_0 be the pressure on the surface of the earth ($z_0=0$) due to a homogeneous atmosphere of density ρ_0 and height H , we have

$$p_0 = g\rho_0 H$$

$$\text{Also } p_0 = K\rho_0$$

$$\therefore K = gH$$

Putting $z_0=0$ and $K = gH$ in (5), we get—

$$\log \frac{p}{p_0} = -\frac{z}{H} \text{ or } p = p_0 e^{-\frac{z}{H}}$$

For a unit increase in z , the changed pressure
 $= p_0 e^{-\frac{(z+1)}{H}} = p_0 e^{-\frac{1}{H}}$.

∴ This shows that as the altitude increases in arithmetical progression, pressure decreases in geometrical progression.

Formula (5) may be used for comparing difference of level by observing barometric pressure. Thus, we have

$$z - z_0 = - \frac{g}{K} \log \frac{P}{P_0}$$

case II when temperature is not constant.

Then by Charles's Law, relation between P and p is

$$p = K_p (1 + \alpha t)$$

Let t_0 be the temperature at the earth's surface and t_1 , at a point P , then the temperature of the atmosphere is their mean, $t = \frac{1}{2}(t_0 + t_1)$. If K' corresponds to this mean temperature, then [Instead of $K(t_0)$]

$$K' = K \left[1 + \alpha \cdot \frac{1}{2}(t_0 + t_1) \right] \quad \text{--- (6)}$$

Hence as in (5), we get —

$$\log \frac{P}{P_0} = - \frac{g}{K'} (z - z_0)$$

$$\Rightarrow z - z_0 = - \frac{K'}{g} \log \frac{P}{P_0}$$

$$= - \frac{K \left[1 + \alpha \cdot \frac{1}{2}(t_0 + t_1) \right]}{g} \log \left(\frac{P}{P_0} \right) \text{ from (6)}.$$

This formula gives height between two stations in terms of the barometric pressure when the temperature is not uniform; but attraction due to gravity, g , is taken to be constant.

case III when the attraction due to gravity is not constant.

Let g be the gravity at sea level and r the radius of the earth. Then by the inverse square law of attraction, the value of gravity g' at a height z above the surface (distance from centre $= r+z$)

is given by

$$g' = \frac{\lambda}{(r+z)^2} \text{ where } g = \frac{\lambda}{r^2} \text{ or } \lambda = gr^2$$

$$\text{so that } g' = \frac{gr^2}{(r+z)^2}$$

Hence The pressure is given by

$$dp = -g'p dz = -\frac{gr^2}{(r+z)^2} p dz, \text{ Also } A = kp.$$

$$\therefore \frac{dp}{p} = -\frac{gr^2}{k^2} \cdot \frac{dz}{(r+z)^2} \quad \left[\because p = kp \right]$$

$$\Rightarrow \log p = \frac{gr^2}{k^2} \cdot \frac{1}{r+z} + C$$

Let $p = p_0$ when $z = z_0$.

$$\therefore \log p_0 = \frac{gr^2}{k^2} \cdot \frac{1}{r+z_0} + C$$

Subtracting,

$$\log \frac{p}{p_0} = \frac{gr^2}{k^2} \left(\frac{1}{r+z} - \frac{1}{r+z_0} \right)$$

To find height above surface of the earth, when $\frac{p}{p_0}$

$$\log \frac{p}{p_0} = \frac{gr^2}{k^2} \left(\frac{1}{r+z} - \frac{1}{r} \right) = -\frac{grz}{k^2(r+z)} \quad \text{--- (7)}$$

p_0 being the pressure on the surface of the earth.

Again if H_0 be the barometric height at the earth's surface and H_1 at the point P , then

$$p_0 = g\sigma_0 (1-\alpha t_0) H_0, \quad \sigma_0 \text{ being density of mercury at } 0^\circ\text{C}, \alpha = \frac{1}{5550}$$

$$\text{and } p = g'\sigma_0 (1-\alpha t_1) H_1 = \frac{gr^2}{(r+z)^2} \sigma_0 (1-\alpha t_1) H_1.$$

$$\text{This gives, } \frac{p_0}{p} = \frac{(r+z)^2}{r^2} \cdot \frac{H_0 (1-\alpha t_0)}{H_1 (1-\alpha t_1)}$$

from (7)

$$z = + \frac{k'}{g} \left(\frac{r+z}{r} \right) \log \frac{p_0}{p} = + \frac{k'}{g} \cdot \log \frac{p_0}{p} \quad \left(\because \frac{z}{r} \text{ is very small} \right)$$

$$= + \frac{k'}{g} \cdot \cancel{\left(\frac{r+z}{r} \right)} \cdot \log \left[\left(\frac{r+z}{r} \right)^2 \frac{H_0 (1-\alpha t_0)}{H_1 (1-\alpha t_1)} \right]$$

$$= + \frac{k'}{g} \log \frac{H_0 (1-\alpha t_0)}{H_1 (1-\alpha t_1)}.$$

$$= - \frac{k'}{g} \left(1 + \frac{\gamma}{\gamma} \right) \log \left[\left(1 + \frac{\gamma}{\gamma} \right)^{\frac{H_0(1-\theta t_0)}{H_1(1-\theta t_1)}} \right]$$

$$= + \frac{k'}{g} \log \frac{H_0(1-\theta t_0)}{H_1(1-\theta t_1)} \quad \text{since } \frac{\gamma}{\gamma} \text{ is small.}$$

$$= + \frac{k' [1 + \frac{1}{\gamma} d(t_0 + t_1)]}{g} \log \frac{H_0(1-\theta t_0)}{H_1(1-\theta t_1)}$$

putting value of k' from (6)

Ques. Absolute Temperature: Let the temperature be lowered, keeping the volume constant; then the pressure will diminish with temperature. Let us imagine the temperature at which the pressure vanishes. This temperature is called the 'absolute zero' of the temperature and absolute temperature is measured from this point. (Scale of absolute temp. is Kelvin.).

The relation between P and t at constant volume

$$\therefore P = kP(1+\alpha t) \quad \text{--- (1)}$$

$$\text{Let } P = 0 \text{ when } t = t_0 \quad \cancel{\text{then}} \quad \cancel{1+\alpha t_0 = 0} \quad \begin{matrix} \text{No being} \\ \text{in } {}^\circ\text{C.} \end{matrix}$$

$$\Rightarrow t_0 = -\frac{1}{\alpha} = -273 \quad \text{and } \alpha = \frac{1}{273}$$

Thus absolute zero is -273°C .

If T be the absolute temperature corresponding to $t^\circ\text{C}$, then $T = t + 273 = t + \frac{1}{2}$

\therefore (1) can be written as

$$P = kP \alpha \left(\frac{1}{2} + t \right)$$

$$= kP \alpha T$$

$$\Rightarrow P = RPT \quad \text{--- (2)}$$

where T is the absolute temperature and R is a constant depending on the nature of the gas.

Since mass of the gas is constant,

$$PV = \text{const}$$

$$\text{or } \frac{P}{RT} V = \text{const} \quad \left[\because P = \frac{P}{RT} \text{ by } \textcircled{1} \right]$$

$$\Rightarrow \frac{PV}{T} = \text{const.} \quad \because R \text{ is const.}$$

Thus if a given mass of gas has pressures, volumes and absolute temperature P, V, T and P', V', T' in two positions, then

$$\frac{PV}{T} = \frac{P'V'}{T'}.$$

Ex ₁₀₇ The readings of a perfect mercucial barometer are α and β , while the corresponding of a faulty one, in which there is some air, are a and b ; prove that the correction to be applied to any reading c of the faulty barometer is

$$\frac{(a+\lambda)(\beta-b)(a-b)}{(a-c)(\lambda-a) - (b-c)(\beta-b)}$$

Soln Let V be the reading of the perfect barometer corresponding to the reading c of the faulty barometer.

If p_1, p_2, p_3 be the pressures of the inside air when the reading in the faulty barometer are a, b and c ; and ρ the density of mercury we have

(Pressure of inside air) + (wt. of the mercury in faulty barometer) = wt. of the mercury in perfect barometer

$$\begin{aligned} \therefore p_1 + \rho g a &= \rho g \alpha \\ p_2 + \rho g b &= \rho g \beta \\ p_3 + \rho g c &= \rho g \delta \end{aligned} \quad \left. \right\} \rightarrow \textcircled{1}$$

Again let A be the cross-section of the barometric tube and l be its length; then when

The barometric reading is a , the volume of the air in the barometer is $(l-a)A$.

But, pressure \times volume = constant

$$\therefore p_1(l-a)A = \text{constant}$$

$$\therefore p_1(l-a) = p_2(l-b) = p_3(l-c)$$

$$\text{or } \rho g(\alpha-a)(l-a) = \rho g(\beta-b)(l-b) = \rho g(\gamma-c)(l-c) \quad (\text{by } ①)$$

$$\text{or } (\alpha-a)(l-a) = (\beta-b)(l-b) = (\gamma-c)(l-c) \quad ②$$

From the first two of these, we get

$$l [(\alpha-a) - (\beta-b)] = a(l-a) - b(l-b)$$

$$\Rightarrow l = \frac{a(l-a) - b(l-b)}{(\alpha-a) - (\beta-b)}$$

Now, the correction applied

$$\begin{aligned} &= \gamma - c \\ &= \frac{(\beta-b)(l-b)}{(l-c)} \quad \text{by } ② \\ &= (\beta-b) \cdot \frac{a(l-a) - b(l-b)}{a(l-a) - b(l-b) - c} \\ &= (\beta-b) \left\{ \frac{a(l-a) - b(\beta-b) - b(l-a) + b(\beta-b)}{a(l-a) - b(\beta-b) - c(l-a) + c(\beta-b)} \right\} \\ &= \frac{(\beta-b)(l-a)(a-b)}{(l-a)(a-c) - (\beta-b)(b-c)} \\ &= \frac{(l-a)(\beta-b)(a-b)}{(a-c)(l-a) - (b-c)(\beta-b)} \quad \text{proved} \end{aligned}$$

Ex 106 If the pressure of the air varies as $(1 + \frac{1}{m})^{\text{th}}$ power of the density, show that, neglecting variations of temperature and gravity, the height of the atmosphere would be equal to $(m+1)$ times the height of the homogeneous atmosphere.

Soln. Let p be the pressure and ρ the density at any point; then as given

$$p = \lambda \rho^{\frac{m+1}{m}} = \lambda \rho^{\frac{m+1}{m}}$$

$$\text{or } p^{\frac{m}{m+1}} = \lambda^{\frac{m}{m+1}} \rho \quad \text{--- (1)}$$

Also, if δ be the height of the point above the surface of the earth, the pressure at this point is given by

$$dp = -\rho [g d\delta] \quad \text{--- (2)}$$

$$(2) \div (1) \Rightarrow \frac{dp}{p^{\frac{m}{m+1}}} = -g \lambda^{\frac{m}{m+1}} d\delta$$

$$\text{Integrating, } \frac{p^{\frac{m}{m+1}+1}}{\frac{m}{m+1}+1} = -g \lambda^{\frac{m}{m+1}} \delta + C$$

$$\left((m+1) \left(\lambda \rho^{\frac{m+1}{m}} \right)^{\frac{m}{m+1}} \right) = (m+1) p^{\frac{1}{m+1}} = C - g \lambda^{\frac{m}{m+1}} \delta$$

$$\Rightarrow (m+1) p \cdot p^{\frac{m}{m+1}} = C - g \lambda^{\frac{m}{m+1}} \delta$$

$$\Rightarrow (m+1) p \cdot \left(\frac{\lambda}{p} \right)^{\frac{m}{m+1}} = C' - g \delta \quad [\text{dividing by } \lambda^{\frac{m}{m+1}}]$$

$$\Rightarrow (m+1) p \cdot \frac{1}{p} = C' - g \delta \quad \text{by (1)}$$

Now, let h_1 be the height of the heterogeneous atmosphere i.e. when $\delta = h_1$, $p = 0$, we have

$$0 = C' - gh_1 \text{ or } C' = gh_1$$

$$\therefore (m+1) \frac{p}{p} = g(h_1 - \delta)$$

If on the surface of earth $p = p_0$, then we have

$$p = p_0, p = p_0 \text{ when } \delta = 0$$

$$\therefore (m+1) \frac{p_0}{p_0} = gh_1$$

$$\text{or } h_1 = \frac{(m+1)p_0}{g p_0} \quad \text{--- (3)}$$

This gives height of the heterogeneous atmosphere when the pressure on the surface of earth is p_0 .

Again if h_2 is the height of the homogeneous atmosphere for a pressure p_0 at the surface of the earth, we have

$$p_0 = \rho_0 g h_2$$

Putting this value of p_0 in (3), we get

$$h_1 = (m+1)h_2$$

✓ This proves the result.

Ex 708 A cylindrical well of depth h and section A is maintained at constant temperature; if ρ_0 and ρ_1 are the densities of the air at the top and bottom, show that the total amount of air contained is

$$\frac{Ah(\rho_1 - \rho_0)}{\log \rho_1 - \log \rho_0}$$

Soln. At depth z , the pressure is given by

$$dp = \rho g dz, \text{ also } p = k\rho$$

$$\therefore \frac{k d\rho}{\rho} = g dz \quad \text{or} \quad \frac{d\rho}{\rho} = \frac{g}{k} dz$$

$$\Rightarrow \log \rho = \log c + \frac{g}{k} z$$

$$\Rightarrow \rho = c e^{\frac{gz}{k}}$$

But when $z=0$, $\rho = \rho_0$. (at the top)

$$\therefore \rho_0 = c$$

$$\therefore \rho = \rho_0 e^{\frac{gz}{k}}$$

Also when, $z=h$, $\rho = \rho_1$ (at the bottom)

$$\therefore \rho_1 = \rho_0 e^{\frac{gh}{k}} \quad \text{--- (1)}$$

$$\therefore \frac{\rho_1}{\rho_0} = e^{\frac{gh}{k}} \quad \text{or} \quad \frac{gh}{k} = \log \frac{\rho_1}{\rho_0}$$

$$\text{or } k = \frac{gh}{\log(\rho_1/\rho_0)} \quad \text{--- (2)}$$

∴ The mass of the liquid contained in the well, area of whose base is A

$$= \int_0^h \rho A dz = A \int_0^h \rho_0 e^{\frac{gz}{k}} dz$$

$$= A P_0 \cdot \left[\frac{k}{g} e^{\frac{gh}{k}} \right]_0^h = - \frac{A P_0 k}{g} (e^{\frac{gh}{k}} - 1)$$

$$\therefore \frac{A P_0}{g} \cdot \frac{gh}{\log(p_1/p_0)} \cdot \left[\frac{p_1}{P_0} - 1 \right] \quad \text{from (1) & (2).}$$

$$= \frac{A h (p_1 - p_0)}{\log p_1 - \log p_0} . //$$

Ex 7/6 A straight tube, closed at one end and open at the other, revolves with a constant angular velocity about an axis meeting the tube at right angles; neglecting the action of gravity, find the density of the air within the tube at any point.

(c) AB is the tube open at A and closed at B, revolving about OZ such that OA = a.

If ω is the angular velocity, the pressure at a point P at a distance r from OZ is

given by

$$dp = \rho \omega r dr; \text{ also } P = kp.$$

$$\therefore \frac{dp}{p} = \frac{\omega}{K} r dr.$$

$$\text{Integrating, } \log p = \log e + \frac{\omega r^2}{2K}.$$

$$\Rightarrow p = C e^{\frac{\omega r^2}{2K}}$$

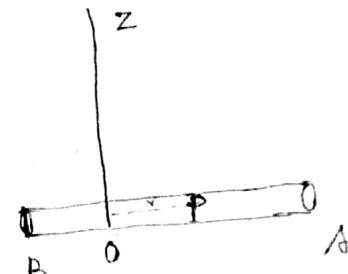
If π be the atmospheric pressure, then at $r=a$, and $p=\pi$, this gives

$$\pi = C e^{\frac{\omega a^2}{2K}}$$

$$\therefore p = \pi e^{\frac{\omega r^2}{2K}(r-a^2)}$$

$$\therefore p = \frac{p}{K} = \frac{\pi}{K} e^{\frac{\omega r^2}{2K}(r-a^2)}$$

which give the density of air at any point.



$$\begin{aligned} \text{Centrifugal force} &= \frac{mv^2}{r} \\ \text{Prism} &= \frac{\rho \cdot v \cdot r l}{2} \\ \therefore dp &= \rho [pdv + rTd\theta + zdz] \\ P, T, Z &\rightarrow \text{component force per unit mass} \\ \text{in } r, \theta, z \text{ directions} \end{aligned}$$