NUCLEAR PHYSICS

BY

DR. UPAKUL MAHANTA,

Asst.Prof., Department of Physics,
Bhattadev University, Bajali

MSc. 2nd Semester Class notes

Contents

Chapter 1

General Properties of Nuclei

1.1 Introduction

Everything we can see in the night time sky is made of nuclear matter. Nuclear physics describes how the Sun generates the energy we need for life on Earth, how all the atoms in your body were made in stars and what happens in stars when they die. Nuclear physics research tries to answer the fundamental questions: **Where Do We Come From? What Are We? Where Are We Going?**
Nuclear physics is the field of physics that studies atomic nuclei and their constituents and interactions. Other forms of nuclear matter are also studied. Nuclear physics should not be confused with atomic physics, which studies the atom as a whole, including its electrons. The history of nuclear physics as a discipline starts with the discovery of radioactivity by Henri Becquerel in 1896 while investigating phosphorescence in uranium salts. In the years that followed, radioactivity was extensively investigated, notably by Marie and Pierre Curie as well as by Ernest Rutherford and his collaborators. By the turn of the century physicists had also discovered three types of radiation emanating from atoms, which they named alpha, beta, and gamma radiation. The 1903 Nobel Prize in Physics was awarded jointly to Becquerel for his discovery and to Marie and Pierre Curie for their subsequent research into radioactivity. Rutherford was awarded the Nobel Prize in Chemistry in 1908 for his "investigations into the disintegration of the elements and the chemistry of radioactive substances". The key experiment behind this announcement was performed in 1910 at the University of Manchester: Ernest Rutherford's team performed a remarkable experiment in which Geiger and Marsden under Rutherford's supervision fired alpha particles (helium nuclei) at a thin film of gold foil. Rutherford's analysis of the data in 1911, led to the Rutherford model of the atom, in which the atom had a very small, very dense nucleus containing most of its mass, and consisting of heavy positively charged particles with embedded electrons in order to balance out the charge. That's where the entire things start developing out.

1.2 Fundamental Forces of Nature:

As you sit on your chair, reading this article, with your laptop or desktop and your android phone nearby, you may be unaware of the many forces acting upon you. A force is defined as a push or pull that changes an object's state of motion or causes the object to deform. Newton defined a force as anything that caused an object to accelerate according to $F = ma$, where F is force, m is mass and a is acceleration. The familiar force of gravity pulls you down into your seat, toward the Earth's center. You feel it as your weight. Why don't you fall through your seat? Well, another force, electromagnetism, holds the atoms of your seat together, preventing your atoms from intruding on those of your seat. The remaining two forces work at the atomic level, which we never feel, despite being made of atoms. The strong force holds the nucleus together. Lastly, the weak force is responsible for radioactive decay, specifically, beta decay where a neutron within the nucleus changes into a proton and an electron, which is ejected from the nucleus. Thus there are four fundamental forces present in nature. Let's now become a bit more technical about these forces

1. Strong force (also known as strong nuclear force:)
The strong interaction is very strong, but very short-ranged (order is 10^{-15} m). It is responsible for holding the nuclei of atoms together. It is basically attractive, but can be effectively repulsive in some circumstances. Mesons are the force carrier for it in case of nucleon (ie protons and neutrons) and Gluons are the force carrier in the case of quarks. Thus even the quarks inside of the protons and neutrons are bound together by the exchange of the strong nuclear force. The relative strength is 1. Time frame for them is 10^{-23} sec. It obeys all the conservation rules. Isospin (a hypothetical concept)is responsible for this force. This force is charge independent, then spin dependent and also it always saturates.

2. Electro-magnetic force:
The electromagnetic force causes electric and magnetic effects such as the repulsion between like electrical charges or the interaction of bar magnets. It is long-ranged, but much weaker than the strong force. It can be attractive or repulsive, and acts only between pieces of matter carrying electrical charge. Electricity, magnetism, and light are

all produced by this force. Relative strength is in the order of 0.01. Time frame for this is 10^{-16} to 10^{-21} sec. All conservation rule are obeyed except the isospin. Charge is responsible for this force. Photons are the force carriers.

3. Weak force:

The weak force is responsible for radioactive decay and neutrino interactions. It has a very short range and. As its name indicates, it is very weak. The weak force causes Beta decay ie. the conversion of a neutron into a proton, an electron and an antineutrino. Relative strength is in the order of 10^{-10}. Time frame for interaction is 10^{-7} to 10^{-10} sec. Many conservation rule is violated. Spin is responsible for this force. Vector bosons (Z^0,W^+,W^-) are the force carriers.

4. Gravitational force:

The gravitational force is weak, but very long ranged. Furthermore, it is always attractive. It acts between any two pieces of matter in the Universe since mass is is responsible for this force. Relative strength is 10^{-40}. Force is mediated by a hypothetical graviton (a spin 2 particle).

1.3 Nuclear terminology:

Nuclei are specified by: **Z**: - atomic number that is the number of protons, **N**: - neutron number that is the number of neutrons, **A**: - mass number that is the number of nucleons, so that A is Z + N. We will also refer to A as the nucleon number. The charge on the nucleus is Ze, where e is the absolute value of the electric charge on the electron. Nuclei with combinations of these three numbers are also called nuclides and are written ${}_ZX^A$ where X is the chemical symbol for the element. Some other common nomenclature is:

Table 1.1: Nuclear terminologies

Word	Definition	Example
Nuclide	A nuclear species	
Isotope	Nuclei with same number of protons	${}_6C^{12}$ & ${}_6C^{13}$
Isotone	Nuclei with same number of neutrons	${}_6C^{12}$ & ${}_7N^{13}$
Isobar	Nuclei with same mass number	${}_6C^{14}$ & ${}_7N^{14}$
Isomer	Nuclei with same number of protons & neutons but with different energies	${}_9F^{19}$ & $[{}_9F^{19}]^\star$

See the star ($\star$) mark actually shows that particular nucleus is at higher energy level than the other.

NOTE: *If you people are finding it difficult to memorize here is a trick. Isoto**p**e. See **p** for proton. Thus same number of protons. Similarly Isoto**n**e. **n** is for neutron, same number of neutrons. Iso**a**r. **a** is for A (mass number) nuclei with same mass numbers. However for Isom**e**r, **e** is for energy but here the energy of the nuclei is different (not same like earlier cases).*

•*Some other important terms*:

• **Atomic Mass Unit (amu)**: An atomic mass unit is defined as precisely $\frac{1}{12}$th the mass of an atom of ${}_6C^{12}$. In imprecise terms, one amu is the average of the proton rest mass and the neutron rest mass (Rest Mass! Go back to your BSc 4th semester's special relativity classes). This is approximately $1\,\text{amu} = 1.66054 \times 10^{-27}$ kg . Now three basic data (! Always remember these three.)

Mass of proton= 1.007276 **amu** **Mass of neutron**= 1.008665 **amu** **Mass of electron**= 0.00055 **amu**

• **Mass Defect**: Mass defect refers to the difference in mass between of a nucleus and the sum of the masses of the protons, neutrons that is its constituent particles. Another alternative way of saying the same thing is the difference between the mass of an isotope and its mass number. Well to be honest and in simple terms what you will found is that the actual mass of the nucleus is always going to be different than the sum of the indivisual mass of the total number of neutrons and protons. To put in a mathematical way

$$\Delta m = (n_{prot}m_{prot} + n_{neut}m_{neut}) - M$$

where 'n's are numbers, 'm's are masses and M is mass of the formed nucleus.

• **Binding energy**: Nuclear binding energy is the minimum energy that would be required to disassemble the nucleus into its component parts. These component parts are ofcourse neutrons and protons. The binding energy is always a positive number, and thus we need to spend energy in moving the nucleons away from each other (attracted by nuclear force). Other way of saying the same thing is that it's the amount of energy that has got utilised while holding the nucleons together inside the nucleus. Now where does this energy come from? Our authority Albert Einstein had the last laugh. His simple innocent looking formula $E = mc^2$, describing the equivalence of energy and mass says that adding energy increases the mass (both weight and inertia), whereas removing energy decreases the mass. It is not advisable to talk about mass being converted to energy or similar expressions. It is better to say that, in measuring an objects mass, we are determining its energy. Suddenly people pointed their fingers towards the mass defect. They said the mass defect times c squared is the energy that holds the nucleus which is nothing but the binding energy. Mathematically

$$\Delta E = \Delta m \times c^2$$

Let's now find how much 1 amu does in MeV corresponds to. This oftens comes in your exam with 2 marks. Here is how we do it. Well obviously using Einstein's formula $E = mc^2$

$$E = mc^2 \,=\, 1.6605 \times 10^{-27}\,kg \times (2.9979 \times 10^8)^2\,(m/sec)^2 \,=\, 15.0639 \times 10^{-11}\,J \,=\, \frac{15.0639 \times 10^{-11}\,J}{1.6022 \times 10^{-13}\,\frac{J}{MeV}} \,=\, 931.5\,MeV$$

As we know that $1.6022 \times 10^{-19}\,J = 1\,eV$. Now paste this in your memory chip. $\boxed{\text{1 amu = 931.5 MeV}}$ Let us take an example for the α-particle which is the ${}_2He^4$ nucleus. This nucleus contains 2 protons and 2 neutrons. Now sum of the masses of the indivisual constituents can be calculated as follows

$$\mathrm{M}' \,=\, 2 \times 1.007276\,\mathrm{amu} \,+\, 2 \times 1.008665\,\mathrm{amu} \,=\, 4.031882\,\mathrm{amu}$$

And the mass (M) of the α-particle is 4.001506 amu. That means we are in a position to calculate the mass defect of the nucleus which is $\Delta m \,=\, 4.031882\,\mathrm{amu} \,-\, 4.001506\,\mathrm{amu} \,=\, 0.030376\,\mathrm{amu}$. Hence the binding energy will be $\Delta\,E \,=\, \Delta m \times 931.5\,\mathrm{MeV/amu} \,=\, 0.030376\,\mathrm{amu} \times 931.5\,\mathrm{MeV/amu} \,=\, 28.29\,\mathrm{MeV}$. Well this is the amount of energy gets released from one single He^4 nucleus. Now if I take 4 gms of He^4 it will contain Avogadro's number of nuclei. Then the release of energy will be $28.29\,\mathrm{MeV} \times 6.023 \times 10^{23}\,Mol^{-1}$ which is almost, after doing a little bit of algebra, 2.56×10^6 mega joules of energy. How tremendous this energy is? Well, it will heat up about 3 million gallons of water from room temperature to boiling point. Can you imagine just about 4 gms of He^4 has the ability to heat up about 2 million gallons of water? Pretty impressive, right?

1.4 Nuclear stability:

Nuclear Stability is a concept that helps to identify the stability of a nuclear species. For example two isotope can have different abundance means their stability is different in different ways. But before we move on let us think about the following question.

Why does every nucleus wants to get stability? Think of yourself when you are tired and ready for sleep. In this case you will most likely just stay put and not do anything as if you don't have anything to spare. The major underlying reason is: "Nature seeks the lowest energy state". In the lowest energy state, things are most stable and less likely to change. One way to view this is that energy makes things happen. If a nucleus is at its lowest energy state, it has no energy to spare to make a change occur. The following information that talks about stability is all based on the nucleus tending towards the lowest energy state. Unstable nuclei will try and become stable by getting to a lower energy state. They will typically do this by emitting some form of radioactivity and change in the process.

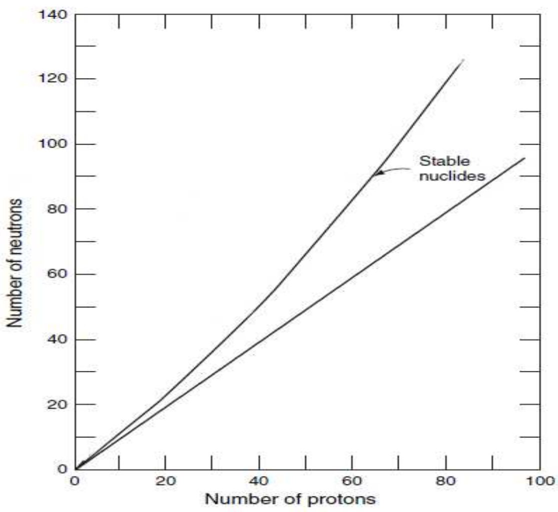
The main factors that determine nuclear stability are

1. the neutron-proton ratio **2.** the packing fraction of the nucleus. **3.** the binding energy per unit nucleon

Let us have some basic ideas about what these are actually.

- **The neutron-proton ratio**:

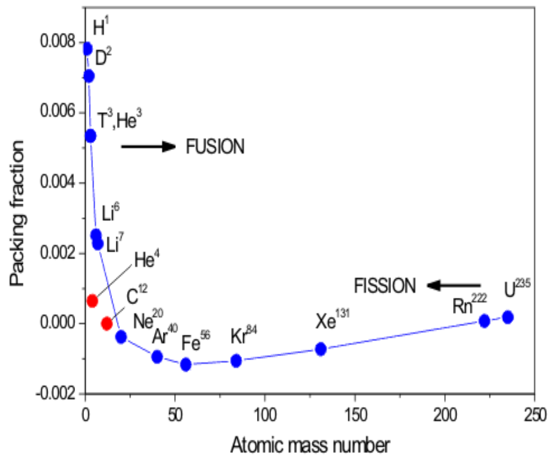
The neutron-proton ratio (N/Z ratio or nuclear ratio) of an atomic nucleus is the ratio of its number of neutrons to its number of protons which is a principal factor for determining whether a nucleus is stable. Elements with ($Z < 20$) are lighter and these elements' nuclei and have a ratio of 1:1 and prefer to have the same amount of protons and neutrons amongst stable and naturally occurring nuclei. But for heavier nuclei this ratio generally increases as the atomic number increases. This is because electrical repulsive forces between protons scale with distance differently than strong nuclear force attractions. In particular, most pairs of protons in large nuclei are far enough, then the electrical repulsion dominates over the strong nuclear force, and thus instability increases. This means if the nucleus has to be still holding up then more number of neutrons will be needed just to give more number of attractive forces in the nuclear core as the neutrons are chargeless. Thus N/Z ratio will become more than 1 for heavier nuclei. The graph in the right side is what I am saying.



- **Packing Fraction**:

It is defined as mass defect per unit nucleon. The value of packing fraction depends upon the manner of packing of the nucleons with in the nucleus. It's value can be negative, positive or even zero. A positive packing fraction describes a tendency towards instability. A negative packing fraction means isotopic mass is less than actual mass number indicates stability of the nucleus. From the figure it is clear that the packing fraction beyond mass number 200 becomes positive and increases with increase in mass number. In general, lower the packing fraction, greater is the binding energy per nucleon and hence greater is the stability. Mathematically it is defined as

$$p_f = \frac{\text{Isotopic Mass} - \text{Mass Number}}{\text{Mass Number}} \times 10^4$$



- **Binding Energy per unit nucleon**:

Well the binding energy curve is obtained by dividing the total nuclear binding energy by the number of nucleons. As simple as that! However the term binding energy, a rather confusing because you might have often thought that this means that energy is required to bind nucleons together. As with chemical bonds, this is the opposite of the truth. Energy is needed to break bonds. But for us it is actually the measure of stability of the nucleus. Larger the binding energy per nucleon, the more stable the nucleus is and the greater the work that must be done to remove the nucleon from the nucleus. The next graph shows the pattern of BE/A for all the nuclei sitting in the periodic table.

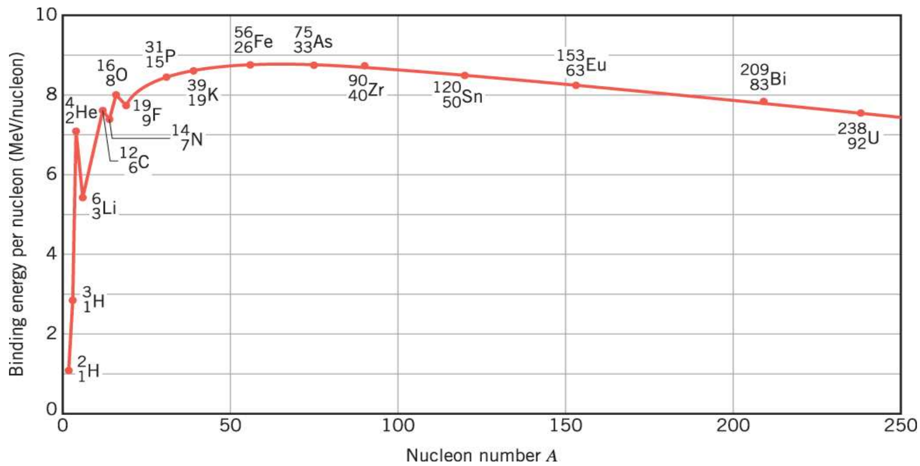


Figure 1.1: Graph of Binding energy per unit nucleon vs mass number.

Important features of the graph:

Few things we can interprete from the above graph which are indeed very important observation. Following are those

1. Excluding the lighter nuclei, the average binding energy per nucleon is about 8 MeV.
2. The maximum binding energy per nucleon occurs at around mass number A = 50, and corresponds to the most stable nuclei. Iron nucleus Fe^{56} is located close to the peak with a binding energy per nucleon value of approximately 8.8 MeV. Its one of the most stable nuclides that exist.
3. Nuclei with very low or very high mass numbers have lesser binding energy per nucleon and are less stable because the lesser the binding energy per nucleon, the easier it is to separate the nucleus into its constituent nucleons.

Q. Explain nuclear fusion from the binding energy curve?

Answer:

The fact that there is a peak in the binding energy curve in the region of stability near iron means that nuclei with low mass numbers may undergo nuclear fusion, where light nuclei are joined together under certain conditions so that the final product may have a greater binding energy per nucleon. Let's have an idea what I mean by that of course with an example.

H^2 has a binding energy of roughly 1.12 MeV per nucleon. Since the reactants in our equation have a total mass of 4 amu, the total binding energy for two H^2 nuclei is: 4 × 1.12 MeV = 4.48 MeV. The product of the reaction, He^4, has a binding energy of roughly 7.08 MeV per nucleon. This gives us a total binding energy of: 4 × 7.08 MeV = 28.32 MeV. Subtracting the initial binding energy from the final binding energy give us: 28.32 MeV - 4.48 MeV = 23.84 MeV which is the amount of energy given off in the fusion. A very destructive indeed....!

Q. Explain nuclear fission from the binding energy curve?

Answer:

Nuclei with high mass numbers may undergo nuclear fission, where the nucleus split to give two daughter nuclei with the release of neutrons. Remember the splitting of Uranium to Barium and Krypton and another three neutrons (HS 2nd year). The daughter nuclei (ie Ba & Kr) will possess a greater binding energy per nucleon as their position will be towardsthe left of the binding energy curve close to Fe^{56}. Thus fission also increases the binding energies of daughter nuclei.

1.4.1 Odd-Even rule of nuclear stability:

We want to know why there is a radioactivity. What makes the nucleus a stable one? There are no concrete theories to explain this, but there are only general observations based on the available stable isotopes. It appears that neutron to proton (N/Z) ratio is the dominant factor in nuclear stability. This ratio is close to 1 for atoms of elements with low atomic number and increases as the atomic number increases. Then how do we predict the nuclear stability? One of the simplest ways of predicting the nuclear stability is based on whether nucleus contains odd/even number of protons and neutrons:

Table 1.2: Odd-Even rule of nuclear stability

Protons	**Neutrons**	**No. of Stable Nuclides**	**Stability**
Odd	Odd	4	least stable
Odd	Even	50	↓
Even	Odd	57	↓
Even	Even	168	most stable

catch of this table:

• Nuclides containing odd numbers of both protons and neutrons are the least stable means more radioactive.
• Nuclides containing even numbers of both protons and neutrons are most stable means less radioactive.
• Nuclides contain odd numbers of protons and even numbers of neutrons are less stable than nuclides containing even numbers of protons and odd numbers of neutrons.

Q. Based on the even-odd rule presented above, predict which one would you expect to be radioactive in each pair?

(a) ${}_8O^{16}$ & ${}_8O^{17}$ (b) ${}_{17}Cl^{35}$ & ${}_{17}Cl^{36}$ (c) ${}_{10}Ne^{20}$ & ${}_{10}Ne^{17}$ (d) ${}_{20}Ca^{40}$ & ${}_{20}Ca^{45}$ (e) ${}_{80}Hg^{195}$ & ${}_{80}Hg^{196}$

Answer:

(a) The ${}_8O^{16}$ contains 8 protons and 8 neutrons (even-even) and the ${}_8O^{17}$ contains 8 protons and 9 neutrons (even-odd). Therefore, ${}_8O^{17}$ is radioactive.

(b) The ${}_{17}Cl^{35}$ has 17 protons and 18 neutrons (odd-even) and the ${}_{17}Cl^{36}$ has 17 protons and 19 neutrons (odd-odd). Hence, ${}_{17}Cl^{36}$ is radioactive.

(c) The ${}_{10}Ne^{20}$ contains 10 protons and 10 neutrons (even-even) and the ${}_{10}Ne^{17}$ contains 10 protons and 7 neutrons (even-odd). Therefore, ${}_{10}Ne^{17}$ is radioactive.

(d) The ${}_{20}Ca^{40}$ has even-even situation and ${}_{20}Ca^{45}$ has even-odd situation. Thus, ${}_{20}Ca^{45}$ is radioactive.

(e) The ${}_{80}Hg^{195}$ has even number of protons and odd number of neutrons and the ${}_{80}Hg^{196}$ has even number of protons and even number of neutrons. Therefore, ${}_{80}Hg^{195}$ is radioactive.

1.5 Nuclear Structure and Dimensions:

The radius of a nucleus is not well defined, since we cannot describe a nucleus as a rigid sphere with a given radius. However, we can still have a practical definition for the range at which the density of the nucleons inside a nucleus approximate our simple model of a sphere for many experimental situations (e.g. in scattering experiments). A simple formula that links the nucleus radius to the mass number is the *empirical radius formula*

$$R = R_0\, A^{\frac{1}{3}}$$

where $R_0 = 1.12\,\text{fm}$ and 1 fm = 10^{-15}m. But from this we actually arrive at a very fundamental conclusion which is

Q. Show that nuclear density is constant for all nuclei?

Answer: We know that

$$R = R_0\, A^{\frac{1}{3}}$$

$$\text{Vol.}\, V = \frac{4}{3}\pi\, R^3 = \frac{4}{3}\pi\, R_0^3\, A = \frac{4}{3}\pi\, R_0^3\, A$$

Therefore density ρ is

$$\rho = \frac{A}{\frac{4}{3}\pi\, R_0^3\, A} = \frac{1}{\frac{4}{3}\pi\, R_0^3}$$

which is constant term. Thus it can be shown that the nuclear density is constant for all nuclei.

1.6 Nuclear Charge Distribution:

When the energy of the incident α-particle energy becomes too large, there is a deviation from the Rutherford scattering formula is observed. The reason for this is that the Rutherford scattering formula was derived assuming that the nucleus was a **point particle**. In reality it has a finite size with a radius R of order 10^{-15} m. The nucleus therefore has a charge distribution, $\rho(r)$. In terms of quantum mechanics we can write

$$\rho(r) \,=\, Ze\,|\Psi(r)|^2 \qquad\qquad (\text{ where symbols have their usual meaning})$$

Nuclear 'radius' is the extent over which the electric charge distribution of the proton, and therefore its wavefunction, is not too small, although in principle the wave-function extends throughout all space. So in order to get a more detailed picture we use **high energy electrons** instead of α-particle to probe the charge distribution. We know the Rutherford's scattering formula where the scattering of α- particles from nuclei can be modeled from the Coulomb force and treated as an orbit. The scattering process can be treated statistically in terms of the cross-section for interaction with a nucleus which is considered to be a point charge Ze. For a detector at a specific angle with respect to the incident beam, the number of particles per unit area striking the detector is given by the Rutherford formula:

$$\text{In terms of scattering cross-section} \quad \frac{d\sigma}{d\Omega} \,=\, \left(\frac{Z\,e^2}{4\,\pi\,\epsilon_0\,K_E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}} \quad \& \quad \text{In terms of no.} \quad N(\theta) \,=\, \frac{N_i\,n\,L\,Z^2\,k^2\,e^4}{4\pi^2\,K_E^2\,\sin^4\frac{\theta}{2}}$$

where θ is the scattering angle, N_i is number of incident α-particle, n is the number of target atoms per unit volume, L is thickness of the target, Z is the atomic number of target atom, k is Coulomb's constant, e charge of the electron and K_E is the kinetic energy of the α-particle. For electrons which are moving faster ie relativistically with a velocity v close to c, there has to be a correction to be introduced and was first calculated by Mott and we have

$$\left.\frac{d\sigma}{d\Omega}\right|_{Mott} \,=\, \left.\frac{d\sigma}{d\Omega}\right|_{Rutherford} \left[1 \,-\, \frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right]$$

The cross section for scattering from a point-like target is given by the Rutherford scattering formula. If the target has a finite spatial extent, the cross section can be divided into two factors, the cross section times the squared of a term called **form factor**, which takes care of the spatial extent and shape of the target. Thus the probability amplitude for a point-like scatterer gets modified by the form factor. So we have

$$\left.\frac{d\sigma}{d\Omega}\right|_{exp} \,=\, \left.\frac{d\sigma}{d\Omega}\right|_{Mott} |F(p^2)|^2$$

here p is the momentum transfered by the electron in the scattering and its magnitude is related to the scattering angle. Thus mathematically the **form factor** is defined as the ratio by which the scattering cross-section is reduced when the charge $+Ze$ is spread out over a finite volume. In Coulomb scattering, the particular property of the spatial extent sampled is the charge distribution $\rho(r)$ for the object for which there will be an potential $V(r)$. Let us now find the expression of the form factor. In order to get to this we will use the first order Born approximation method *(Recall your last semester quantum mechanics class)*. To first order (and up to a normalization) the Born Approximation the scattering amplitude can be written as

$$\begin{aligned}
f^{1st}_{Born} &= \langle \Psi_f(k)\,|\,V(r)\,|\Psi_i(k)\,\rangle \\
&= \int \Psi_f^\star(k)\,V(r)\,\Psi_i(k)\,d^3r \,=\, \int e^{-ik_f r}\,V(r)\,e^{ik_i r}\,d^3r \qquad \text{assuming plane waves } e^{ikr} \\
&= \int V(r)\,e^{i(k_i-k_f)r}\,d^3r \,=\, \int V(r)\,e^{\frac{ipr}{\hbar}}\,d^3r
\end{aligned}$$

This result is still quite general; in order to proceed we will need to assume a specific form for the potential, $V(r)$. We can describe an extended charge distribution by $Ze\,\rho(r)$ with $\int \rho(r)\,d^3r \,=\, 1$ and since we are dealing with a coulomb potential ie the potential experienced by an electron located at r' is given by

$$V(r) \,=\, -\frac{Z\,e^2}{4\,\pi\,\epsilon_0}\int \frac{\rho(r)}{|r\,-\,r'|}\,d^3r'$$

Substituting this potential into the general expression for the first Born Approximation to the scattering amplitudes we get

$$f^{1st}_{Born} \,=\, -\frac{Z\,e^2}{4\,\pi\,\epsilon_0}\int \frac{\rho(r)}{|r\,-\,r'|}\,d^3r' \int e^{\frac{ipr}{\hbar}}\,d^3r$$

Now making the substitution $r - r' = R$ and $d^3R = d^3r$

$$f^{1st}_{Born} = -\frac{Z\,e^2}{4\,\pi\,\epsilon_0} \int \frac{\rho(r')}{|R|}\,d^3r' \int e^{\frac{ip(R+r')}{\hbar}}\,d^3R$$
$$= -\frac{Z\,e^2}{4\,\pi\,\epsilon_0} \int \frac{e^{\frac{ipR}{\hbar}}}{|R|}\,d^3R \left[\int \rho(r')\,e^{\frac{ipr'}{\hbar}}\,d^3r'\right]$$

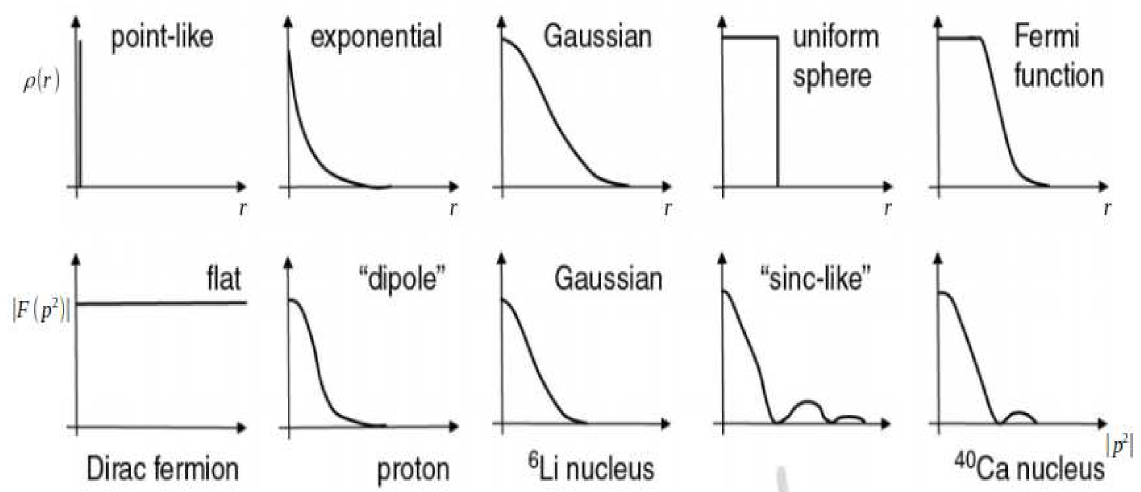
Qualitatively, this can be interpreted as that the part of the wavefront that passes through the nucleus at a distance r from the centre and is scattered through an angle θ travels a further distance than the part of the wave that passes through the centre, by an amount proportional to r' and therefore suffers a phase change. This phase change also depends on the scattering angle θ and is equal to $\frac{pr'}{\hbar}$. This means that different parts of the wavefront suffer a different phase change (just as in optical diffraction) these different amplitudes are summed to get the total amplitude at some scattering angle θ and this gives rise to the diffraction pattern. The contribution to the amplitude from the part of the wavefront which passes at a distance r from the centre of the nucleus is proportional to the charge density, $\rho(r')$ at r'. The total scattering amplitude is therefore the sum of the amplitudes from all these different parts, which is what the last integral means. Hence the bracked factor in the last equation is known as the 'Form Factor' $F(p^2)$ and is defined by this integral over the volume of the target which is nothing but just the **Fourier transform** of the charge distribution of course in 3D. Since the Coulomb's potential is spherically symetric therefore the form factor can be further simplified into

$$F(p^2) = \frac{4\,\pi\,\hbar}{Zep} \int_0^\infty r'\,\rho(r')\sin\left(\frac{pr'}{\hbar}\right)\,dr' \qquad \text{with } d^3r' = r'^2\,dr'\,\sin\theta d\theta\,d\phi$$

So an inverse Fourier transform of the form factor is then going to give us the charge distribution. Note here that the form factor will be zero for a case when $\sin\left(\frac{pr'}{\hbar}\right) = 0$ ie $\frac{pr'}{\hbar} = \pi$. From here we can make a rough estimate of the distance where the charge distribution changes from the order of its value at the centre to zero giving an approximate nuclear radius of about 3 fm. To be this value the charge distribution $\rho(r)$ should take a form given by

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{\delta}\right)}$$

where ρ_0 is the charge density at the center, R as the nuclear 'radius' and δ as the 'surface depth' which measures the range in r over which the charge distribution changes from the order of its value at the centre to much smaller than this value. This is known as the **Fermi distribution**. Sometimes also known as the **Saxon-Woods model** for charge distribution.



1.7 Nuclear Spin

A bit of History

In 1922, at the University of Frankfurt (Germany), Otto Stern and Walther Gerlach, did fundamental experiments in which beams of silver atoms were sent through inhomogeneous magnetic fields to observe their deflection. These experiments demonstrated that these atoms have quantized magnetic moments that can take two values. Although consistent with the idea that the electron had spin, this suggestion took a few more years to develop.

Pauli introduced a " **two-valued**" degree of freedom for electrons, without suggesting a physical interpretation. Kronig suggested in 1925 that it this degree of freedom originated from the self-rotation of the electron. This idea was severely criticized by Pauli, and Kronig did not publish it. In the same year Uhlenbeck and Goudsmit had a similar idea, and Ehrenfest encouraged them to publish it. They are presently credited with the discovery that the electron has an intrinsic spin with value " **one-half**". Much of the mathematics of spin one-half was developed by Pauli himself in 1927. It took in fact until 1927 before it was realized that the Stern-Gerlach experiment did measure the magnetic moment of the electron. These all you have studied in your BSc atomic physics.

Mathematical foundation of Spin

The mathematical theory people developed while describing a quantum state by vectors is that the state vector represents all the information we can know about the system and we used the state vectors to calculate probabilities. With each observable we associated a pair of kets corresponding to the possible measurement results of that observable. The observables themselves are not yet included in our mathematical theory, but the distinct association between an observable and its measurable kets provides the means to do so. The role of physical observables in the mathematics of quantum theory is described by the following postulates listed below.

Postulate 1: Physical states are represented by mathematical vectors or **kets** say $|\psi\rangle$.

Postulate 2: A physical observable is represented mathematically by an operator A that acts on kets say $A|\psi\rangle$.

Postulate 3: The only possible result of a measurement of an observable is one of the eigenvalues λ of the corresponding operator A is $A|\psi\rangle = \lambda|\psi\rangle$.

Postulate 4: An operator is always diagonal and eigenvectors are unit vectors in their own basis.

Postulate 5: After the measurement of A that yields the result λ, the quantum system is in a new state that is the normalized projection of the original system ket onto the ket (or kets) corresponding to the result of the measurement $|\phi\rangle = \dfrac{A|\psi\rangle}{\langle\psi|A|\psi\rangle}$

While dealing with spin it's a bit awkward to picture the wavefunctions for electron spin because the fermions (electrons, protons or neutrons) aren't spinning in normal 3D space, but in some internal dimension that is "rolled up" inside them. So physicists have invented some abstract states "α" and 'β" that represent the two possible orientations of the spin, but because there isn't a classical analog for spin we can't draw "α" and 'β" wavefunctions. So in the abstract way we will be manipulating operators and wavefunctions without looking explicitly at what the wavefunction or operator looks like in real space. The wonderful tool that we use to do this is called **Matrix Mechanics** (as opposed to the wave mechanics we have been using so far). We will use the simple example of spin to illustrate how matrix mechanics works.

The basic idea of matrix mechanics is then to replace the wavefunction with a vector which is not a vector in physical (x, y, z) space but just a convenient way to arrange the coefficients that define the wavefunction. Now, our goal is to translate everything that we might want to do with the wavefunction say $\Psi(s)$ into something we can do to the vector $\Psi(s)$. By going through this stepbystep, we arrive at a few rules in matrix mechanics. Let us first define $\Psi(s)$ as

$$\Psi(s) = c_1\,\alpha + c_2\,\beta \qquad \rightarrow \qquad \Psi(s) = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Now any the overlap between any two wavefunctions can be written as a modified dot product between the vectors. So on the similar note $\Phi(s)$ is as

$$\Phi(s) = d_1\,\alpha + d_2\,\beta \qquad \rightarrow \qquad \Phi(s) = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

So

$$\begin{aligned}
\int \Phi^\star(s)\,\Psi(s)\,dV &= \int (d_1\,\alpha + d_2\,\beta)^\star\,(c_1\,\alpha + c_2\,\beta)\,dV = \int (d_1^\star\,\alpha^\star + d_2^\star\,\beta^\star)\,(c_1\,\alpha + c_2\,\beta)\,dV \\
&= d_1^\star\,c_1 + d_2^\star\,c_2 \qquad\qquad \text{as } \alpha^\star\,\beta = \beta^\star\,\alpha = 0 \quad \text{and} \quad \alpha^\star\,\alpha = \beta^\star\,\beta = 1 \\
&= (d_1^\star\ d_2^\star)\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \Phi^\dagger\,\Psi
\end{aligned}$$

So **integrals are replaced with dot products** in matrix mechanics. Also $(d_1^\star\ d_2^\star) = \Phi^\dagger = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}^\dagger$ is nothing but taking **the adjoint of the vector** $\Phi(s)$. Finally, we'd like to be able to act operators on our states in matrix mechanics, so that we can compute average values, solve eigenvalue equations, etc. We know that in wave mechanics operators turn a wavefunction into another wavefunction. Thus, in order for operators to have the analogous behavior in matrix mechanics, operators must turn vectors into vectors. As it turns out this is the most basic property of a matrix: it turns vectors into vectors. So in matrix mechanics **operators are represented by matrices.**

But before going into the mathematics part let me give you a gereral remark on a quantum state which you have probably felt during your first semester. The state of a quantum system is modeled as a unit-length element $|\Psi\rangle$ of a complex Hilbert space $\mathcal{H}$, a special kind of vector space with an inner product. Every observable quantity (like momentum or spin) associated with such a system whose value one might want to measure is represented by a self-adjoint operator on that space. If one builds a device to measure such an observable, and if one uses that device to make a measurement of that observable on the system, then the machine will output an eigenvalue λ of that observable. Moreover, if the system is in a state $|\Psi\rangle$, then the probability that the result of measuring that quantity will be the eigenvalue of the observable is $|\langle\lambda|\Psi\rangle|^2$

Specialization to spin systems, The S-operator

Suppose, now, that the system we are considering consists of the spin of a particle. The Hilbert space that models the spin state of a system with spin s is a $2s+1$ dimensional Hilbert space. Elements of this vector space are often called **"spinors"**. The cartesian components of the spin (which is an observable) of the system are three self-adjoint operators conventionally called S_x, S_y and S_z, as **spin matrices** whose eigenvalues are the possible values one might get if one measures one of these components of the system's spin. More explicitly, since protons and neutrons are spin-$\frac{1}{2}$ system, and one chooses to represent states and observables in a basis consisting of the normalized eigenvectors of the z component of spin, then one would find the following matrix representations in that basis

$$S_x = \frac{\hbar}{2}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = \frac{\hbar}{2}\sigma_x \qquad S_y = \frac{\hbar}{2}\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix} = \frac{\hbar}{2}\sigma_y \qquad S_z = \frac{\hbar}{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \frac{\hbar}{2}\sigma_z$$

where σ_x, σ_y and σ_z are called as the **Pauli spin matrices** or **Pauli spinors**. These spinors have the follwoing properties

$$\mathrm{Tr}\,[\sigma_i] = 0 \qquad |\sigma_i| = -1 \qquad \text{and} \qquad \sigma_x\,\sigma_y\,\sigma_z = i$$

Since you know that **sum of the eigenvalues of a matrix is equal to the trace of a matrix** here a trace-less matrix would indicate that the sum of the eigenvalues of the matrix must be zero. Thus you get spins are $\pm\frac{1}{2}$. Also a trace-less matrix would indicate a **commutator**. Since spin is a type of angular momentum, it is reasonable to suppose that it possesses similar properties to orbital angular momentum. Thus we can arrive at the same conclusion that

$$[S_x\,S_y] = i\,\hbar\,S_z \qquad [S_y\,S_z] = i\,\hbar\,S_x \qquad [S_z\,S_x] = i\,\hbar\,S_y$$

In the similar note we can also have the matrix algebra of the Pauli spinors. Let's just a look at it.

$$\sigma_x\,\sigma_y = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix} = i\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = i\,\sigma_z$$

If you reverse the order ie $\sigma_y\,\sigma_x$ will become $-i\,\sigma_z$. This in turn will lead to general conclusion that

$$\begin{aligned}\sigma_x\,\sigma_y &= i\,\sigma_z = -\,\sigma_y\,\sigma_x\\ \sigma_y\,\sigma_z &= i\,\sigma_x = -\,\sigma_z\,\sigma_y\\ \sigma_z\,\sigma_x &= i\,\sigma_y = -\,\sigma_x\,\sigma_z\end{aligned}$$

which have retained the its cyclic behaviour. So immediately we can have

$$\sigma_x\,\sigma_y - \sigma_y\,\sigma_x = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix} - \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = 2\,i\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = 2\,i\,\sigma_z$$

This result can be generalised as $[\sigma_i\,\sigma_j] = 2\,i\,\sigma_k$ which have also retained its cyclic behaviour. Here we dfine a new object called **commutator**. Mathematically speaking the commutator of two operators is defined as the difference between the products of the two operators taken in alternate orders. Thus we have found that the Pauli spinors which **do not commute** since the value of the commutator is not zero. Had it been zero then $AB - BA = 0$ which will mean $AB = BA$. Thus for commuting operators, the order of operation does not matter, which then further will lead us to the fact that these two operators will have then **simultaneous sets of eigenstates**. Thus we say that we can know the eigenvalues of these two observables **simultaneously**. It is common to extend this language and say that these two observables can be measured **simultaneously**, though, we do not really measure them simultaneously. What we mean is that we can measure one observable without erasing our knowledge of the previous results of the other observable. Observables A and B are said to be **compatible**. Thus this is straight cut violation of Heisenberg's uncertainty principle. But for non-commuting operators these all will not happen. Thus when it comes to spin the conclusion to draw from this is that while we can know one spin component absolutely (ie $\Delta\,S_z = 0$), we can never know all three, nor even two, simultaneously. This lack of ability to measure all spin components simultaneously implies that the spin does not really point in a given direction, as a classical spin or angular momentum does. So when we say that we have measured "spin up," we really mean only that the spin component along that axis is up, as opposed to down, and not that the complete spin angular momentum vector points up along that axis.
Also it can be easily seen that

$$\sigma_x\,\sigma_y + \sigma_y\,\sigma_x = \sigma_y\,\sigma_z + \sigma_z\,\sigma_y = \sigma_z\,\sigma_x + \sigma_x\,\sigma_z = 0$$

This is known as "**anti-commuatation**", i.e., not only do the spin operators not commute amongst themselves, but the anticommute! They are strange beasts.

The S^2-operator

Another indication that the spin does not point along the axis along which you measure the spin component is obtained by considering a new operator that represents the magnitude of the spin vector but has no information about the direction. It is common to use the square of the spin vector for this task. This new operator is

$$\begin{aligned} S^2 &= S_x^2 + S_y^2 + S_z^2 \\ &= \left(\frac{\hbar}{2}\right)^2 \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^2 + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}^2 + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^2\right] = 3\left(\frac{\hbar}{2}\right)^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \propto \mathcal{I} \end{aligned}$$

Thus the S^2 operator is proportional to the identity operator, which means it must commute with all the other operators S_x, S_y and S_z ie $[S^2,\, S_i] = 0$. It also means that all states are eigenstates of the S^2 operator.

1.8 Nuclear Parity

Parity is a rather subtle concept and has no classical analogue. It is concerned with the behaviour of wave function under space inversion. Two kinds of parity actually correspond to two different kinds of quantum wave function for a particle. Nuclear states have a well defined parity, defined by the behaviour of the wavefunction for all the nucleons under reversal of their coordinates with the centre of the nucleus at the origin. This is equivalent to studying the mirror image of the original system. It was originally assumed that parity must be conserved in all particle interactions, but it was demonstrated that parity does not have to be conserved in β- decay. Parity of nuclear ground states can usually be determined from the shell model which be dealt in coming chapters.

$$\Psi\,(-x,\,-y,\,-z) = \lambda\,\Psi\,(x,\,y,\,z)$$

The parity operator Π is defined as $\Pi\,|x\rangle = |-x\rangle$ which is a Hermitian such that $\Pi^\dagger = \Pi$. So the eigen value operation is given by from definition

$$\begin{aligned} \Pi\,|\Psi\rangle &= \lambda\,|\Psi\rangle \\ \Pi^2\,|\Psi\rangle &= \lambda^2\,|\Psi\rangle \\ \langle\Psi\,|\Pi^2\,|\Psi\rangle &= \lambda^2\,\langle\,\Psi|\Psi\,\rangle \\ \langle\Psi|\Psi\rangle &= \lambda^2\,\langle\,\Psi|\Psi\,\rangle \\ \lambda^2 &= 1 = \lambda \pm 1 \end{aligned}$$

Thus if x represents space then we have $\langle x\,|\Pi\,|\Psi\rangle = \langle -x|\Psi\rangle = \Psi(-x) = \lambda\Psi(x)$. The parity is eventually

$$\Psi(x) = \begin{cases} \text{even} & \lambda = 1 \\ \text{odd} & \lambda = -1 \end{cases}$$

• **Symmetric wave function:** Wave functions for which the value at the point $(-x,\,-y,\,-z)$ is the same as at the point $(x,\,y,\,z)$ are known as symmetric wave functions.
• **Antisymmetric wave function:** Wave functions for which the value at the point $(-x,\,-y,\,-z)$ is minus the value as at the point $(x,\,y,\,z)$ are known as antisymmetric wave functions.

1.9 Nuclear Isospin

Let me just draw an analogy to introduce the concept of "**Isospin**". We already know that electrons have two spin values with respect to the z-direction. ie $S_z = \pm\frac{1}{2}$ which then can be distinguished by the application of an external non-uniform magnetic field in the z-direction. But in the absence of this external field these two cannot be distinguished and we are used to thinking of these as two states of the same particle. So we need to invoke the principle of superposition to describe the state of the electronic spin.
Similarly, if we could 'switch off' electromagnetic interactions we would not be able to distinguish between a proton and a neutron. Also as far as the strong interactions are concerned it is also charge independent. So we just can't distinguish protons and neutrons as a charged and neutral particle in nuclear physics. Thus then, these are just two states of the same particle (a nucleon). Then how will we distinguish between them? The answer is by **isospin**. What we therefore think of an imagined space in which the nucleon has this property which is mathematically analogous to spin but has nothing to do with angular momentum. The proton and neutron are now considered to be a nucleon with different values of the third component of this isospin I_3 or sometimes I_z. This isospin is associated with a conservation law which requires strong interaction decays to conserve isospin. This term was derived from isotopic spin, but physicists prefer the term isobaric spin, which is more precise in meaning.
Since this third component can take two possible values, we assign $I_3 = \frac{1}{2}$ for the proton and $I_3 = -\frac{1}{2}$ for the

neutron. Therefore nucleon has isospin $I = \frac{1}{2}$ in the same way that the electron has spin $s = \frac{1}{2}$, with two possible values of the third component.
In the case of nucleons the electric charge, Q is related to the third component of isospin by

$$Q = I_3 + \frac{1}{2}$$

You just put the values of third component of isospin you will get the answer why proton is positively charged and neutron is chargeless. Other particles can also be classified as isospin multiplets. For example there are three pions, π^+, π^0 and π^-, which have almost the same mass and zero spin etc. There are three of them with different charges but which behave in the same way under the influence of the strong interactions. Thus they are nothing but three different states of the same particles called pions. However their charges can't be explained on the basis of the above formula. Hence the formula needs modification. Now let us draw a head to head comparison between two electronic states and two nucleonic states.

<u>Two electronic states</u>
Two electrons can have a total spin $S = 0$ or $S = 1$. The total wavefunction is then

$$\begin{aligned} \Psi_{12} &= \Psi(r_1, r_2)\, \chi_s(s_1, s_2) \\ &= \text{(spatial part) (spin part)} \end{aligned}$$

For $S = 1$ we have the spin part as

$$\begin{aligned} \chi_s(s_1, s_2) &= (\uparrow\uparrow) & S_z &= 1 \\ &= \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) & S_z &= 0 \\ &= (\downarrow\downarrow) & S_z &= -1 \end{aligned}$$

which is symmetric under interchange of the two spins, which means that by fermi statistics the spatial part of the wavefunction must be antisymmetric under the interchange of the positions of the electrons.

For $S = 0$ we have the spin part as

$$\chi_s(s_1, s_2) = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

which is antisymmetric under interchange of spins so it must be accompanied by a symmetric spatial part of the wavefunction.

<u>Two nucleonic states</u>
Two nucleons can have a total isospin $I = 0$ or $I = 1$. The total wavefunction is then

$$\begin{aligned} \Psi_{12} &= \Psi(r_1, r_2)\, \chi_s(s_1, s_2)\, \chi_I(I_1, I_2) \\ &= \text{(spatial part) (spin part) (isospin part)} \end{aligned}$$

For $I = 1$ we have the isospin part as

$$\begin{aligned} \chi_I(I_1, I_2) &= (p\,p) & I_3 &= 1 \\ &= \frac{1}{\sqrt{2}}(pn + np) & I_3 &= 0 \\ &= (n\,n) & I_3 &= -1 \end{aligned}$$

which is symmetric under the interchange of the isospins, so that it must be accompanied by a combined spatial and spin wavefunction that must be antisymmetric under simultaneous interchange of the two positions and the two spins.
For $I = 0$ we have the isospin part as

$$\chi_I(I_1, I_2) = \frac{1}{\sqrt{2}}(pn - np)$$

which is antisymmetric under the interchange of the two isospins and therefore they must be accompanied by a combined spatial and spin wavefunction which is symmetric under simultaneous interchange of the two positions and the two spins.

Chapter 2

Nuclear β-decay

2.1 Introduction

In the last chapter we spoke about stability of nuclei. But not all the nuclei are stable. What does that lead to? This chapter will focus on what the unstable nuclei will do? Well it's just simple. They will decay.
Because the nucleus experiences the intense conflict between the two strongest forces in nature, it should not be surprising that there are many nuclear isotopes which are unstable and emit some kind of radiation. Radioactive decay (also known as **nuclear decay** or **radioactivity**) is the process by which an unstable atomic nucleus loses energy by emitting radiation, such as an α particle, β particle or γ particle. To put it in another way the atomic nuclei that dont have enough binding energy to hold the nucleus together due to an excess of either protons or neutrons are going to disintegrate.
Let's have some history. Radioactivity was discovered in 1896 by the French scientist Henri Becquerel, while working with phosphorescent materials. These materials glow in the dark after exposure to light, and he suspected that the glow produced in cathode ray tubes by X-rays might be associated with phosphorescence. He wrapped a photographic plate in black paper and placed various phosphorescent salts on it. All results were negative until he used uranium salts. The uranium salts caused a blackening of the plate in spite of the plate being wrapped in black paper. It soon became clear that the blackening was also produced by non-phosphorescent salts of uranium and metallic uranium. These radiations were given the name "**Becquerel Rays**".

2.2 Properties of Radioactivity:

The modes and characteristic energies that comprise the decay scheme for each radioisotope are specific. If instrumentation is sufficiently sensitive, it is possible to identify which isotopes are present in a sample. But that will cost lot of your money. Radioactive decay will change one nucleus to another if the product nucleus has a greater nuclear binding energy than the initial decaying nucleus. The difference in binding energy (comparing the before and after states) determines which decays are energetically possible and which are not. But let me put all the information about radioactivity in a straight forward form.
• It is entirely a nuclear phenomenon is due to the instability of the nucleus.(*Remember the N/Z ratio*)
• It is a spontaneous and irreversible process. (*Well that's obviously the thing going to be, once it's emitted it's emitted, you can never rerun it back.*)
• It is independent of external factors such as pressure, temperature, state of substance, electrical field, magnetic field, catalyst etc.(*Take a sample, push it, shake it do anything you want to do! The nucleus will show extreme disrespect to your activity*)
• A radioactive element emits α, β or γ radiations which probabilistic in nature and does not depend on the age of the nucleus or how it was created. (*You never can predict when a certain nuclei is going to emit a particle.*)
• A radioactive element does not emit α and β particles simultaneously.(*Two bullets are fired from a gun simultaneously right! Well that's impossible. Common sense.*)
• The original radioactive nucleus or element is called a parent element and the new element formed is known as daughter element. (*That's the terminolgy we use*)
• It's a first order reaction.(*means it will need infinite time to finish*)
• The physical and chemical properties of daughter element are different than that of the parent element.(*I will tell you later why this is so*).

2.2.1 Artificial and Natural Radioactivity

The harsh reality is that radioactivity has not been invented by man; it has been there, existing in the universe since time immemorial. The of nuclei which takes place in nature, is called natural radioactivity. However there are elements beyond uranium which have been artificially made. They are called the transuranium elements which can be made to disintegrate into other nuclei by colliding with slow moving neutrons. This is called artificial radioactivity. Thus it is customery to check the difference between these two types. Well there may be a question in your exam from this

Natural Radioactivity	**Artificial Radioactivity**
1. Radioactivity that takes place on its own in nature	1. It is induced by man in laboratories
2. Occurs in elements with $Z > 82$	2. Can be induced in elements with low Z.
3. It usually have long half life.	3. This usually have short half life.
4. Decay particles are α, $+\beta$ & γ	4. Decay particles are α, $\pm\beta$, & γ

table during your BSc. But I request you to remember these as this is also probabilistic in nature...may or may not come in your MSc. exam.

2.2.2 Properties of α, β & γ-decay

In radioactive processes, particles or electromagnetic radiation are emitted from the nucleus. The most common forms of radiation emitted have been traditionally classified as α, β, and γ radiation. Let's now inspect the characteristics of them.

• **Characteristics of α-decay**
1 These particles are helium nuclei $_2He^4$.(*Alpha rays consist of stream of positively charged particles carrying charge of* $+2$ *units and a mass almost equal to* 4 *amu*)
2 They affect photographic plate
3 They are deflected only slightly towards the negative plate in electric field. They are also deflected by magnetic field. (*see they are charged and hence Lorentz force is in action*)
4 These particles can ionize gases. Alpha rays have maximum ionizing power. (*Again because these particles will interact with the medium as they are charged*)
5 They have a velocity of the order of $1 \times 10^7 ms^{-1}$.
6 They have very little penetrating power.(*mass is the culprit in this case*)

• **Characteristics of β-decay**
1 Beta rays are electrons $_{-1}e^0$.(*these rays are made up of streams of negatively charged particles with a negligible mass.*
2 They affect photographic plate.
3 They get deflected to the maximum extent towards the positive plate in electric field. They are also deflected by magnetic field.(*Again Lorentz force*)
4 Their ionising power is less than that of α- rays. (*It is about one hundredth of* α- *particles*).
5 Their velocity varies with the source sometimes reaches $2.7 \times 10^8 ms^{-1}$.
6 Their penetration power is about 100 times more than that of α- particles. (*Since mass is too small*).

• **Characteristics of γ-decay**
1 They are electromagnetic radiations (photons) like X-rays having very short wavelength, in the range of 10^{-10} m to 10^{-13} m.
2 They affect photographic plate.
3 They are unaffected by electric and magnetic fields.(*No charge no Lorentz force*)
4 Their ionizing power is low, and is about one hundredth of β- particles.(*No charge no ionisation*)
5 Their velocity is same as that of light.
6 Their penetrating power is very high, about 100 times more than that of β - particles.(*since they donot interact they keep on moving moving... and moving*)

In addition, there are a couple of less common types of radioactive decay, these are as follows:

• **Positron emission**
Although positron emission doesnt occur with naturally occurring radioactive isotopes, it does occur naturally in a few man-made ones. A positron is essentially an electron that has a positive charge instead of a negative charge. A positron is formed when a proton in the nucleus decays into a neutron and a positively charged electron. The positron is then emitted from the nucleus.

• **Electron capture or K-capture**
Electron capture is a rare type of nuclear decay in which an electron from the innermost energy level is captured by the nucleus. This electron combines with a proton to form a neutron. The atomic number decreases by one, but the mass number stays the same. The capture of the 1s electron leaves a vacancy in the 1s orbitals. Electrons drop down to fill the vacancy, releasing energy in the X-ray portion of the electromagnetic spectrum.

2.3 Theory of β-decay

β-particles are either electrons or positrons that are emitted through a certain class of nuclear decay associated with the weak force which is characterized by relatively lengthy decay times. The name β, followed naturally as the next letter in the Greek alphabet after α, α-particles having already been discovered and named by Rutherford. But as we know that the radioactivity is entirely a nuclear phenomenon then where does this e^- come from? But can you remember that the neutron has a larger mass than the proton and is thus unstable with respect to the combination of a proton and an electron. So consider the following

$$
{}_1n^0 \rightarrow {}_1p^1 + {}_0e^{-1}
$$
$$
{}_1p^1 \rightarrow {}_1n^0 + {}_0e^{+1}
$$

Thus inside the nucleus if these things happens it will result in a production of an e^-. Aha..! We now know that there can be an e^- production. But then again **why does that produced e^- comes out of the nucleus. I mean why it can't stay inside the nucleus?** To answer this we need to do a little bit of algebra using some celebrated principles of physics. Next is how we can show that.
We know the Heisenberg's uncertainty principle as $\Delta x \quad \Delta p \geq \frac{h}{2\pi}$. Take Δx as positional uncertainty which is equal the to typical nuclear dimension means the e^- can be anywhere inside the nucleus. Thus $\Delta x = 10^{-15}$m. Mass of the e^-=9.1×10^{-31}kg. Now do a calculation.

$$
\begin{aligned}
\Delta x \, \Delta p &= \frac{h}{2\,\pi} \\
\Delta x \, m \Delta v &= \frac{h}{2\,\pi} \\
\Delta v &= \frac{h}{2\,\pi\, m\, \Delta x} \\
&= \frac{6.63 \times 10^{-34}}{2 \times 3.1415 \times 9.1 \times 10^{-31} \times 10^{-15}} \\
&= 1.2 \times 10^{11} \quad m\,\mathrm{sec}^{-1}
\end{aligned}
$$

That's velocity at which the e^- has to stay inside the nucleus which is straightway violating the Special Theory of Relativity according to which nothing can have a velocity greater the velocity of light. Thus Heisenberg's uncertainty principle along with STR will speak about why can't an e^- reside inside nucleus. Hence the e^- has to come out of course.

2.4 Three forms of β-decay and their conditions for occuring:

Proton decay, neutron decay, and electron capture are three ways in which protons can be changed into neutrons or vice-versa; in each decay there is a change in the atomic number, so that the parent and daughter nuclei are different. In all three processes, the number A of nucleons remains the same, while both proton number, Z, and neutron number, N, increase or decrease by 1. So far so good! Now let's get somewhat detailed into that.

- **β^-- decay:**

In β^-- decay, the weak interaction converts an atomic nucleus into a nucleus with atomic number increased by one, while emitting an electron (e^-) and an electron antineutrino ($\bar{\nu_e}$). β^-- decay generally occurs in neutron-rich nuclei. The generic equation is:

$$
{}_ZX^A \rightarrow {}_{Z+1}Y^A + e^- + \bar{\nu_e}
$$

where A and Z are the mass number and atomic number of the decaying nucleus, and X and Y are the initial and final elements, respectively. Inside the nucleus following is what that has happened.

$$
{}_0n^1 \rightarrow {}_1p^1 + e^- + \bar{\nu_e}
$$

2.4.1 Condition for occurence of β^-- decay:

In β- decay, the mass difference between the parent and daughter particles is converted to the kinetic energy of the daughter particles. This kinetic energy is of course coming from masses of atoms involved in process. Though the atomic mass is almost comparable with the nuclei but still there is minute difference since in case atom the electrons have to also taken into account and they also contribute to the mass. So we must concentrate the nuclear mass rather than the atomic mass. Since neutrinos are massless therefore neglecting it in the equation

$$
{}_ZX^A \rightarrow {}_{Z+1}Y^A + e^-
$$

the disintegration energy Q can be written down as

$$\begin{aligned} Q &= [\text{Nuclear} \ \ \text{mass}\,(X)] - [\text{Nuclear} \ \ \text{mass}\,(Y) + m_{e^-}] \times c^2 \\ &= [\text{Atomic} \ \ \text{mass}\,(\text{X}) - Zm_{e^-}] - [\text{Atomic} \ \ \text{mass}\,(Y) - (Z+1)m_{e^-} + m_{e^-}] \times c^2 \\ &= [M_X - Zm_{e^-} - M_Y + Zm_{e^-} + m_{e^-} - m_{e^-}] \times c^2 \\ &= [M_X - M_Y] \text{ in energy units} \end{aligned}$$

Thus for Q> 0 you must have M$_X > M_Y$. Or to put it in a sentence *"for β^--decay to occur the mass of parent atom must be greater than that of the daughter atom."*

- **β^+- decay:**

In β^-- decay, the weak interaction converts an atomic nucleus into a nucleus with atomic number decreased by one, while emitting a positron (e$^+$) and a electron neutrino (ν_e). β^+- decay generally occurs in neutron-rich nuclei. The generic equation is:

$$_ZX^A \ \rightarrow \ _{Z-1}Y^A \ + e^+ \ + \nu_e$$

where A and Z are the mass number and atomic number of the decaying nucleus, and X and Y are the initial and final elements, respectively. Inside the nucleus following is what that has happened.

$$_1p^1 \ \rightarrow \ _0n^1 \ + e^- \ + \nu_e$$

2.4.2 Condition for occurence of β^+- decay:

Similar treatment I am going to use. We will see the disintegartion energy pertaining to this decay. And will find out the condition. Hence in the equation

$$_ZX^A \ \rightarrow \ _{Z-1}Y^A \ + e^+$$

the Q value of the reaction is

$$\begin{aligned} Q &= [\text{Nuclear} \ \ \text{mass}\,(X)] - [\text{Nuclear} \ \ \text{mass}\,(Y) + m_{e^-}] \times c^2 \\ &= [\text{Atomic} \ \ \text{mass}\,(X) - Zm_{e^-}] - [\text{Atomic} \ \ \text{mass}\,(Y) - (Z-1)m_{e^-} + m_{e^-}] \times c^2 \\ &= [M_X - Zm_{e^-} - M_Y + Zm_{e^-} - m_{e^-} - m_{e^-}] \times c^2 \\ &= [M_X - M_Y - 2m_{e^-}] \text{ in energy units} \end{aligned}$$

Thus for Q> 0 you must have M$_X > M_Y + 2m_{e^-}$. Or to put it in a sentence *"for β^--decay to occur the mass of parent atom must be greater than that of the daughter atom by at least twice the electronic mass."*

- **K-capture:**

This is a process during which a nucleus captures one of its atomic electrons, resulting in the emission of a neutrino. Most commonly the electron is captured from the innermost, or K, shell of electrons around the atom; for this reason, the process often is called K-capture. Here the atomic number decreases by one unit, and the mass number remains the same like positron emission. The generic equation is:

$$_ZX^A \ + e^- \ \rightarrow \ _{Z-1}Y^A \ + \nu_e$$

where A and Z are the mass number and atomic number of the decaying nucleus, and X and Y are the initial and final elements, respectively.

2.4.3 Condition for occurence of K-capture:

Here also the process is going to be same. But one thing is different in this case. See the electron was orbiting before it was getting captured by the nucleus. So it was as if pulled working against the binding energy of the electron in the orbit. So that energy has to be taken into account. The Q value of the reaction is

$$\begin{aligned} Q &= [\text{Nuclear} \ \ \text{mass}\,(X) + m_{e^-}] - [\text{Nuclear} \ \ \text{mass}\,(Y)] \times c^2 - B_e \\ &= [\text{Atomic} \ \ \text{mass}\,(X) - Zm_{e^-} + m_{e^-}] - [\text{Atomic} \ \ \text{mass}\,(Y) - (Z-1)m_{e^-}] \times c^2 - B_e \\ &= [M_X - Zm_{e^-} + m_{e^-} - M_Y + Zm_{e^-} - m_{e^-}] \times c^2 - B_e \\ &= [M_X - M_Y] \times c^2 - B_e \end{aligned}$$

Thus for Q> 0 you must have M$_X > M_Y + B_e$. Or to put it in a sentence *"for K-capture to occur the mass of parent atom must be greater than that of the daughter atom by at least the binding energy of the electron."*

2.4.4 Apparent violation of conservation of Energy: β-decay energy release

Take the following nuclear transmutation

$$ {}_1n^0 \rightarrow {}_1p^1 + {}_0e^{-1} $$

The energy release is given by the following equation

$$
\begin{aligned}
{}_1n^0 &\rightarrow {}_1p^1 + {}_0e^{-1} \\
Q &= [m_n - (m_p + m_{e^-})] \times 931.5 \quad \text{MeV} \\
&= [1.0086 - (1.0072 + 0.00055)] \times 931.5 \;=\; 0.8384 \quad \text{MeV}
\end{aligned}
$$

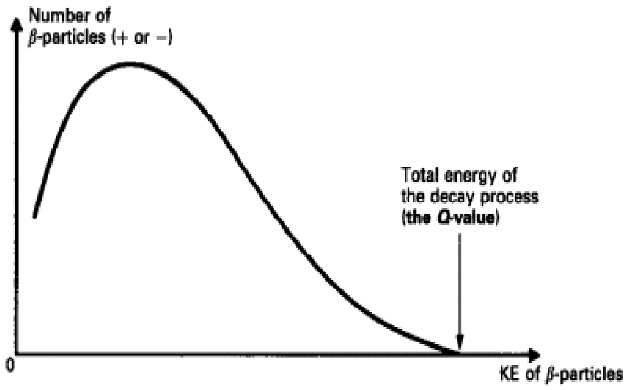
This is the amount of energy with which the e$^-$ comes out of the nucleus. But to big surprise nuclear physicists have found that everytime a nucleus of the same atom undergoes a β-decay the the energy of the e$^-$ is not the same. There is a variation of the energy of the e$^-$ which extends from a maximum at the Q-value down to zero. What happened to the law of conservation of energy for β-decay? This observation has made the physicist to point finger to the principle conservation of energy. They even thought that the principle conservation of energy is a bogus statement or this principle is not valid at least in case of β-decay. A mortal sin for a physicist.

2.4.5 β-decay energy spectrum: Pauli's neutrino hypothesis

Wolfgang Pauli came with a bold statement in order to save the law of conservation of energy, that ${}_1n^0 \rightarrow {}_1p^1 + {}_0e^{-1}$ is wrong transmutation equation. He argued on the basis of spin conservation which is a quantum number. What he said is the following

All the three particles in this equation are fermions with intrinsic spins $s = \pm\frac{1}{2}$. The spins of the proton and the electron can be coupled to either 0 or 1 or -1. This simple spin algebra will never yield the half integral value on the left hand side of the equation. Therefore, we cannot balance the angular momentum in the reaction as written. So the conclusion is another fermion must be present among the products with zero charge and zero mass, a third body could then take away whatever energy was not given to the beta particle; solving that most vexing of issues. Pauli first proposed this hypothesis in a humorous letter to his colleagues Lise Meitner and Hans Geiger. But Enrico Fermi, the great Italian physicist, was immediately convinced and gave a name "neutrino" (meaning a neutral little one in Italian). At the Solvay Conference on October 1933, he proposed the theory of β-decay based on a hypothesis that an electron-neutrino pair is spontaneously produced by a nucleus in the same way that photons can spontaneously be emitted by excited atoms.

Ok! So here we go. A third particle has been proposed to be emitted in the β-decay process as the neutrino which is of course difficult to detect. But then that particle if detected will rescue the the conseravtion of energy principle from breaking down. At the same time it will also explain why everytime the β-particle comes out with different energies. There has been a sharing of energy between the e$^-$. If the e$^-$ is detected to have high energy than the neutrino is going to take away less energy and vice versa. And the curve in your right side is readily explained. And here physicists have have introduced a technical term. The **End point energy** which is defined as the maximum energy carried out by the emitted e$^-$.



2.5 The *Poltergeist* Project: Cowan & Reines Experiment

The path to discovery

• In 1934 Enrico Fermi establishes the theory of weak decay, providing a framework for neutrinos.

• In 1935 Hans Bethe calculates the probability detecting a neutrino experimentally. The reaction cross-section he found is as $\sigma = 10^{-44}$ cm^2.

In fact Cowan and Reines explored Bethe's hypothesis to use inverse β- decay to detect neutrinos. Neutrino remained a hypothetical particle until evidence for its existence was brought forward by them in 1956 in the project which was named as *Poltergeist* because of its illusive properties. There are two interesting episodes connected to the Cowan-Reines experiment.

• In that period (1945-55) many nuclear bomb tests were being conducted. In the explosion of the nuclear bomb also, Uranium nucleus fissions and antineutrinos are produced. Cowan and Reines had planned to catch those antineutrinos, but were prevented from pursuing that dangerous venture. They then changed their plan and went to the Savannah River Reactor (USA) to do their experiment and succeeded.

• Pauli had apparently sent a cable telegram to the Committee which was to decide on the sanction of financial support for the Cowan-Reines experiment, saying that "*his particle*" cannot be detected by anybody and so asking the Committee not to support such an experiment. However that telegram did not reach the Committee in time; support was given and the antineutrino was caught in the experiment!

The principle

In the mid 1950s the nuclear reactors were were put into operation for social welfare. Cowan and Reines had the idea to take advantage of the intense flux of neutrinos they generated, fluxes ranging from 1000 to 10,000 billion neutrinos per second per square centimetre, much more intense than those expected from radioactive sources. In fact every nuclear reactor is a copious source of antineutrinos. When nuclei such as Uranium fission in the nuclear reactor, a variety of radioactive nuclei are produced. Many of them undergo beta decay and emit antineutrinos. Cowan and Reines used a hydrogenous material as their detector. Hydrogen nucleus is a proton. If the antineutrino from the reactor interacts with the proton, a positron and a neutron are produced.

$$\bar{\nu_e} + {}_1p^1 \rightarrow {}_0n^1 + e^+$$

Reines and Cowan proved the appearance of the positron and neutron in their detector placed near the nuclear reactor. So, positron and neutron are proved. On that score the neutrino, in fact antineutrinos from the nuclear reactor was experimentally is verified by Cowan and Reines.

The set-up

Their experimental set up was like the following
• Uranium fission reactor 1000 MW *(Why? It is in order to have that intense neutrino flux so that interaction becomes more.)*
• Two tanks of diluted cadmium chloride ($CdCl_2$) in water sandwiched between *(Why? The positrons, quickly found electrons with which they annihilate in a very characteristic manner through the emission of γ photons of 0.51 MeV enegy. But Cowan and Reines realized that this signature was not enough to prove that the positron was due to a antineutrino interaction. They looked for the presence of the neutron, that accompanies Cd is a efficient neutron absorber used in reactor control rods. By absorbing a neutron, Cd^{108} turns into an excited Cd^{109} nucleus, which emits a characteristic desexcitation γ ray.)*

$$\bar{\nu_e} + {}_1p^1 \rightarrow {}_0n^1 + e^+ \qquad \text{detection}$$
$$ {}_0n^1 + {}_{48}Cd^{108} \rightarrow {}_{48}Cd^{109} + 3\,\gamma \qquad \text{detection}$$

• Three tanks of liquid scintillator, $183 \times 132 \times 56$ cm^3 each. *(Why? The recently discovered organic liquid scintillators are used in response to gamma rays. These scintillators produce flashes of light that were amplified and detected by photomultipliers placed on both sides of the tank.)*
• The apparatus surrounded with thick layer of earth and metal *(Why? It is just to shield it from other particles coming from the reactor and cosmic rays as much as possible.)*

Their findings

• The first series of measurements: 200 hours, 567 events, ~ 200 estimated to be from background.
• Counting rate of ~ 3 events per hour. So they accumulated data during several months.
• The experimental set-up was designed in such a way that the third detected gamma should be detected less than 5 millionth of a second after the two gamma coming from the positron annihilation. The detection of three gamma within such a short time interval was an unmistakable signature of a neutrino interaction. They checked that these events disappear when the reactor was stopped. Finally, they measured for this "inverse β reaction" a rate compatible with the theoretical predictions made by Bethe. And everything fits! Neutrinos finally detected (it took 26 years) ! In 1995 Reines receives the Nobel Prize (However Cowan deceased by that time). *One can only wonder why it took them so long.....* Thus the equation for β-decay becomes

$$ {}_1n^0 \rightarrow {}_1p^1 + {}_0e^{-1} + \bar{\nu_e}$$

2.6 Fermi's theory of β-decay

We are familiar with charged particles that produce (create) an e.m. field. Using the principles of quantum field theory, an e.m. field can be described by as an operator that can create (or destroy) photons to which nobody objected because of the fact that these are massless particles. Since photons are also particles, and by analogy we can have also creation of other types of particles, such as the electrons and the neutrinos but here the difference is that now they are massive. Thus in the light of the possibilities of creating and annihilating particles, we also need to find a new description for the particles themselves that allows these processes. All of this is obtained, of course again, by **quantum field theory** and **the second quantization**. Quantum field theory gives a unification of e.m. and weak force (electro-weak interaction) with one coupling constant e.

Basic assumptions made by Fermi:
- It is a three body decay scheme.
- The mass of the ν is zero.
- There is no recoil of the daughter nucleus.
- The νs and e^-s are assumed to be relativistic but spin less.
- There is no electromagnetic interactions between the e^- and the daughter nucleus. *(later on it was taken care of)*
- The wavelength for the νs and e^-'s motion are significantly larger than the size of the parent/daughter nuclei.

The theory:

The properties of beta decay can be understood by studying its quantum-mechanical description via Fermi's Golden rule

$$\lambda = \frac{2\pi}{\hbar} \left| \left\langle \Psi_f \,\middle|\, U_{int} \,\middle|\, \Psi_i \right\rangle \right|^2 \rho\,(E_f)$$

where U_{int} is a potential that causes the transition from an initial quantum state Ψ_i (the parent nucleus in the this case) to a final one, Ψ_f, that includes wavefunctions of the daughter nucleus, the electron and its neutrino. The matrix element $\left\langle \Psi_f \,\middle|\, U_{int} \,\middle|\, \Psi_i \right\rangle$ gives you the transition amplitude, square of which is the transition probability. And the term $\rho\,(E_f)$ is the density of final states which what is concern for all in order to have the β-spectrum and end-point energy. So our aim is to find out the values of the matrix element and the density of states for both the electron and its neutrino.

Calculation of the matrix element $\left\langle \Psi_f \,\middle|\, U_{int} \,\middle|\, \Psi_i \right\rangle$

As this β-decay is a process of creation of two particles which must be through some interaction U_{int} which can be written in terms of the particle field wavefunctions

$$U_{int} = g\, \Psi_e^\dagger\, \Psi_\nu^\dagger$$

where Ψ_x ($\Psi_x^\dagger$) annihilates (creates) the particle x, and g is the coupling constant that determines how strong the interaction is. So the matrix element can be written as

$$\begin{aligned} U_{if} &= \left\langle \Psi_f \,\middle|\, U_{int} \,\middle|\, \Psi_i \right\rangle = g \left\langle \Psi_f \,\middle|\, \Psi_e^\dagger\, \Psi_\nu^\dagger \,\middle|\, \Psi_i \right\rangle \\ &= g \int \Psi_f^\star \,[\Psi_e^\star\, \Psi_\nu^\star]\, \Psi_i\, d^3r \qquad \text{for scalar operators } \dagger \to \star \end{aligned}$$

To first approximation the electron and neutrino can be taken as plane waves. Why? As they are produced and comes out of the nucleus they will no longer remain under the influence of a potential. Hence in principle they are free. So the wave function corresponding to them is going to be a plane wave. So the plane wave solution corresponding to the electron can be written as $A\, e^{ik_e \cdot r}$ and $B\, e^{ik_\nu \cdot r}$. These wave functions can not be normalized, the normalization constants A and B are infinite if the particles are allowed to propagate into the infinite distances. To avoid this problem we assume that the system is enclosed withing the volume V which can be large but finite. In such cases we can have the normalised plane wave solution for the electron as $\dfrac{e^{ik_e \cdot r}}{\sqrt{V}}$ and for the neutrino it is $\dfrac{e^{ik_\nu \cdot r}}{\sqrt{V}}$. Putting these in the last equation we get

$$U_{if} = g \int \Psi_f^\star \,\frac{e^{-ik_e \cdot r}}{\sqrt{V}}\, \frac{e^{-ik_\nu \cdot r}}{\sqrt{V}}\, \Psi_i\, d^3r$$

The electron and neutrino wavefunctions have wavelengths that are many times the size of the nucleus. This means for both $k \cdot r \ll 1$. Now if we expand these wavefunctions in a Taylor series expansion then $e^{-ik \cdot r} = 1 - (ik \cdot r) - \dfrac{(k \cdot r)^2}{2!} \,\ldots\ldots \approx 1$. So the last expression then boils down to

$$\begin{aligned} U_{if} &= \frac{g}{V} \int \Psi_f^\star\, \Psi_i\, d^3r \\ &= \frac{g}{V}\, M_{if} \end{aligned}$$

This M_{if} is a very complicated function of the nuclear spin and angular momentum states. If $M_{if} \neq 0$ then the transition is called as an **allowed** transition, and the rate is relatively prompt. However if $M_{if} = 0$ then it is called **forbidden** transitions. In such cases we have to go to higher order terms in the approximation and then chances of occurrence is there, but at a much slower rates. One more thing that you should observe here is that for a given decay, M_{if} is constant so that the observed spectrum is proportional to the density of final states ie $\rho(E_{if})$. But so far in deriving theory we have made a big approximation in ignoring the charge on the emitted electron. Positively charged β-particles (positrons) will be repelled by the nucleus and shifted to higher energies while negatively charged β-particles (electrons) will be attracted and slowed down. These effects were incorporated by Fermi by using Coulomb-distorted wave functions and are contained in a spectrum distortion expression called the Fermi function, $F(Z_d,\, p_e)$, where Z_d is the atomic number of the daughter nucleus and p_e is the momentum of the emitted electron. So finally the matrix expression becomes

$$U_{if} = \frac{g}{V}\, M_{if}\, F(Z_d,\, p_e)$$

Calculation of the density of states, $\rho(E_f)$

The last factor, $\rho(E_f)$, is the density of states that are available to the system after the transition. It is generally defined as the number of available states per energy ie $\rho(E_f) = \dfrac{dN_s}{dE_f}$ where N_s is the number of states. But there can be more than one state share the same energy means states can be degenrate too. What it quantum mechanically means is that there may be more than one eigenfunction sharing the same eigenvalues. As in this case there are two particles namely e^- and ν are present and also both can be in a continuum of possible states, the density of states will be equal to the number of electron states per unit energy times the number of neutrino states per unit energy at the fixed energy of the decay ie

$$dN_s = dN_e \, dN_\nu$$

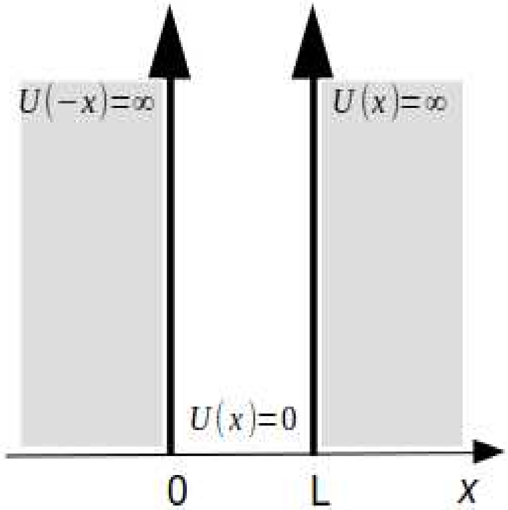
The conservation of energy demands the energy of the neutrino for a fixed energy of the electron has to be constant ie if the two particles share a Q amount of energy then $E = T_e + T_\nu$ where Ts are the kinetic energies. So, in principle we need to find out the energies of these two particles indivisually. As the particles were assumed to be relativistic we can write for the ν

$$T_\nu = \sqrt{p_\nu^2 \, c^2 + m_{0\nu}^2 \, c^4} = p_\nu \, c \qquad \text{(since } \nu\text{s are massless)}$$

$$p_\nu = \frac{E - T_e}{c} \qquad \text{implies} \, dp_\nu = \frac{dE}{c}$$

while for the e^- the kinetic energy will be $T_e = \sqrt{p_e^2 \, c^2 + m_0^2 \, c^4} - m_0 \, c^2$

Now we are going to calculate the number of electron and neutrino states. As that the decaying system (nucleus + emitted particles) is enclosed by the volume $V = L^3$, which means it is embedded withing the infinitely deep three dimensional potential well. But for simplicity let us take an 1D potential well. The normalised wave function $\Psi(x)$ for a particle in 1D and the energy eigen value is given by



$$\Psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n \, \pi \, x}{L}\right) \qquad E = \frac{n^2 \, \pi^2 \, \hbar^2}{2 \, m \, L^2}$$

Now we will generalise it to 3D. So for an infinitely deep 3D potential well the wave function is

$$\Psi(r) = \sqrt{\frac{8}{L^3}} \sin\left(\frac{n_x \, \pi \, x}{L}\right) \sin\left(\frac{n_y \, \pi \, y}{L}\right) \sin\left(\frac{n_z \, \pi \, z}{L}\right)$$

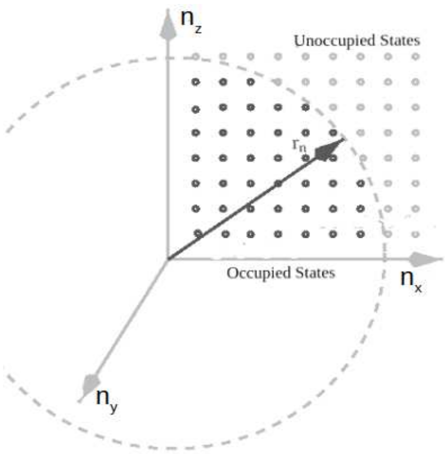
And the energy eigen value is given by

$$E = \frac{\pi^2 \, \hbar^2}{2 \, m \, L^2} \left(n_x^2 + n_y^2 + n_z^2\right) = \frac{n^2 \, \pi^2 \, \hbar^2}{2 \, m \, L^2} = \frac{p_e^2}{2 \, m}$$

$$p_e^2 = \frac{n^2 \, \pi^2 \, \hbar^2}{L^2} \qquad \text{implies} \quad n^2 = \frac{p_e^2 \, L^2}{\pi^2 \, \hbar^2}$$

$$2 \, n \, dn = \frac{L^2}{\pi^2 \, \hbar^2} \, 2 \, p_e \, dp_e \qquad \text{implies} \quad dn = \frac{L}{\pi \, \hbar} \, dp_e$$

Now we can count the number of electron's states in a spherical shell between n and $n + dn$ to be



$$dN_e = 4 \, \pi \, n^2 \, dn = 4 \, \pi \, \frac{p_e^2 \, L^2}{\pi^2 \, \hbar^2} \, \frac{L}{\pi \, \hbar} \, dp_e = 4 \, \pi \, \frac{V}{\pi^3 \, \hbar^3} \, p_e^2 \, dp_e$$

If we consider just a small solid angle $d\omega$ instead of 4π then requires that we take $\dfrac{1}{8}$th of above equation. Hence finally we have then the number of state

$$dN_e = \frac{1}{8} \, 4 \, \pi \, \frac{V}{\pi^3 \, \hbar^3} \, p_e^2 \, dp_e = \frac{V}{2 \, \pi^2 \, \hbar^3} \, p_e^2 \, dp_e$$

In the same notion the number of neutrino's states in a spherical shell between n and $n + dn$ to be $dN_\nu \;=\; \dfrac{V}{2\,\pi^2\,\hbar^3}\,p_\nu^2\,dp_\nu$. So the number of states for the system is then going to be

$$
\begin{aligned}
dN_s \;=\; dN_e\,dN_\nu \;&=\; \frac{V}{2\,\pi^2\,\hbar^3}\,p_e^2\,dp_e\,\frac{V}{2\,\pi^2\,\hbar^3}\,p_\nu^2\,dp_\nu \\
&=\; \frac{16\,\pi^2\,V^2}{h^6}\,p_e^2\,dp_e\,p_\nu^2\,dp_\nu \\
&=\; \frac{16\,\pi^2\,V^2}{h^6}\,p_e^2\,dp_e\left(\frac{Q\,-\,T_e}{c}\right)^2\frac{dQ}{c} \;=\; \frac{16\,\pi^2\,V^2}{h^6\,c^3}\,(Q\,-\,T_e)^2\,p_e^2\,dp_e\,dQ \\
\rho(E_f) \;=\; \frac{dN_s}{dE} \;&=\; \frac{16\,\pi^2\,V^2}{h^6\,c^3}\,(E\,-\,T_e)^2\,p_e^2\,dp_e
\end{aligned}
$$

Thus we arrive at the celebrated equation of Fermi's theory of β-decay, once we replace U_{if} and $\rho(E_f)$ in the equation of λ which was

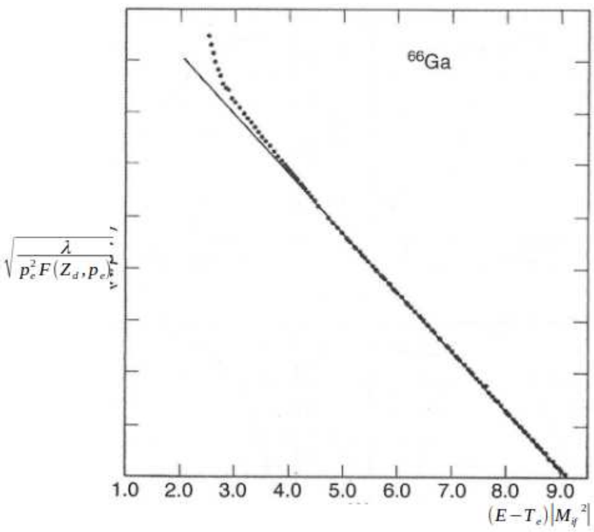
$$
\begin{aligned}
\lambda \;&=\; \frac{2\,\pi}{\hbar}\,\big|\langle\,\Psi_f\,|\,U_{int}\,|\Psi_i\,\rangle\big|^2\,\rho\,(E_f) \\
&=\; \frac{2\,\pi}{\hbar}\,\left(\frac{g}{V}\,M_{if}\,F(Z_d,\,p_e)\right)^2\,\frac{16\,\pi^2\,V^2}{h^6\,c^3}\,(E\,-\,T_e)^2\,p_e^2\,dp_e \\
&=\; \frac{2\,\pi}{\hbar}\,\left(\frac{g}{V}\right)^2\,\big|\,M_{if}\big|^2\,F^2(Z_d,\,p_e)\,\frac{16\,\pi^2\,V^2}{h^6\,c^3}\,(E\,-\,T_e)^2\,p_e^2\,dp_e \\
&=\; \frac{1}{2\,\pi^3\,\hbar^7\,c^3}\,g^2\,\big|\,M_{if}\big|^2\,F^2(Z_d,\,p_e)\,(E\,-\,T_e)^2\,p_e^2\,dp_e
\end{aligned}
$$

Notice that these distributions (as well as the decay rate below) are the product of three terms:
− the Statistical factor (arising from the density of states calculation) $(E\,-\,T_e)^2$
− the Fermi function (accounting for the Coulomb interaction), $F(Z_d,\,p_e)$
− and the Transition amplitude from the Fermi Golden Rule, $\big|U_{if}\big|^2$

These three terms reflect the three ingredients that determine the spectrum and decay rate of in beta decay processes. So the Fermi's equation is nothing but a distribution with a kind of **statistical phase space factor** for the three body decay related to the spectrum of the emitted beta particles (electron or positron).

2.7 Kurie Plot

Suppose we want to determine the Q-value of Ga-66 decaying into Zn. The electrons and neutrinos from this spontaneous decay will share the Q-value as kinetic energy. The neutrinos cannot be detected, but we can measure the kinetic energy of the electrons and receive an energy spectrum. We can in principle determine the Q-value from the spectrum; the highest recorded energy. But the count rate near the end point energy is small due to noise and background and the limited resolution of the detector, so the value cannot be determined from the spectrum with high enough accuracy. To calculate a better approximation of the Q-value from the electron count rate, the Fermi Theory is used to construct a Kurie plot (also known as a **Fermi-Kurie plot**), developed by Franz Kurie, giving us a linear function, which can be extrapolated.



The Kurie plot is the plot of $\dfrac{\lambda}{p_e^2\,F(Z_d,\,p_e}$ placed in y-axis against $(E\,-\,T_e)\,\big|M_{if}\big|^2$ plotted in x-axis. So, basically, a Kurie plot is a graphic means of comparing theoretical and observed momentum distributions in continuous beta-ray spectra. The significance of this coordinates is in the fact that the electron spectrum near the endpoint is linear if y is plotted as a function of x. If the Kurie plot is not straight, one must successively test shape factors until a straight line match is obtained. Once the shape factor is determined, the level of forbiddeness is determined, and the Q-value may be extrapolated from the data unambiguously.

Chapter 3

Models of Nuclei

3.1 Introduction

See, the structure of atoms is now well understood (*Bohr's theory for monoelectronic atom and Hartree-Fock or Sommerfeld's theory for polyelectronic atoms*): quantum physics governs all; the electromagnetic force is the main force; each atom contains a massive central force (the nucleus) that dominates. But in case of the nucleus quantum mechanics still governs its behavior, but

• The forces are complicated and there is no exact mathematical expression that accounts for the nuclear force, in fact, can't be written down explicitly in full detail like electromagnetic or gravitational force.

• The nuclues is actually a many-body problem of great complexity and there is no mathematical solution to the many-body problem.

So, in the absence of a comprehensive nuclear theory, we turn to the construction of nuclear models. Thus nuclear models, are amongst the several theoretical descriptions of the structure and function of atomic nuclei or in other words it is simply a way of looking at the nucleus that gives a physical insight into as wide a range of its properties as possible. The usefulness of a model is tested by its ability to provide predictions that can be verified experimentally in the laboratory. It should be mentioned that each of the models is based on a plausible analogy that correlates a large amount of information and enables predictions of the properties of nuclei. What is that mean is the following.. You observe some properties of a nucleus and as according to that you device a model which will only describe those behaviour. You prepare a different model to explain some other properties. Well the former model is inadequate to describe the properties explained by the later one and like that.

Nuclear models can be classified into two main groups. In those of the first group, called **strong-interaction, or statistical models**, the main assumption is that the protons and neutrons are mutually coupled to each other and behave cooperatively in a way that reflects the short-ranged strong nuclear force between them. The **liquid-drop model and compound-nucleus model** are examples of this group. In the second group,, called **independent-particle models**, the main assumption is that little or no interaction occurs between the individual particles that constitute nuclei; each proton and neutron moves in its own orbit and behaves as if the other nuclear particles were passive participants. The **shell model** and its variations fall into this group. Other nuclear models incorporate aspects of both groups, such as the **collective model** put forwarded by Aage Bohr (son of Neils Bohr), which is a combination of the shell model and the liquid-drop model. The **Optical model** is however one specific model however where the nucleus is assumed as a medium having complex refractive index.

As far as your syllabus is concerned we will mainly look at **liquid-drop** and **shell model** of the nucleus and a bit of Collective model.

3.2 Liquid Drop model of the nucleus

The liquid drop model in nuclear physics treats the nucleus as a drop of incompressible nuclear fluid of very high density. It was first proposed by George Gamow along with Weizsacher in 1935 who have recognized some experimental evidences and found resemblance of nucleus with a liquid drop and then developed by Niels Bohr and John Wheeler later on. What they have justified in favour of this model are the following

• Like the molecules in a drop of liquid, the nucleons are imagined to interact strongly with each other.

• Just like liquid molecules can collide with each other due to thermal agitation but then well inside the drop, a given nucleon collides frequently with other nucleons in the nuclear interior, its mean free path as it moves about being substantially less than the nuclear radius.

• The liquid drop is assumed as imcompressible meaning its density can't be changed similar is the case for nucleus also where the density of the nucleus is constant for all the nuclei.

• The liquid drop is spherical because of surface tension similarly the nucleus is spherical because of the strong nuclear force.

• In case of the liquid drop the cohesive force always saturates just like the nuclear force which also saturates.

• The heat of vaporization which represents the amount of energy required to convert molecules from liquid phase to gas phase or rather more specifically the latent heat of vaporisation is proportional to the number of molecules in the liquid just like the bindind energy of nucleus is also proporsonal to number of nucleon.
However there are some differences too which are as follows
• The nucleus has a limited number of particles (< 270) compared to chemical systems ($\approx 10^{23}$). The net result is that there is a much larger fraction of nucleons on the surface relative to those in the bulk for nuclei compared to chemical systems.
• The nucleus is a two-component system composed of neutrons and protons whereas in a liquid drop number of components may be more or less.
This is a crude model that does not explain all the properties of the nucleus, but does predict the nuclear binding energy. As the model justifies the similarities between a liquid drop and a nucleus one can then construct a semiempirical model (half theory/half data) also known as **Bethe-Weizacker Semi-empirical Mass Formula** to account for the total nuclear binding energy, the most basic of nuclear properties.

The Model : Bethe-Weizacker Semi-empirical Mass Formula (SEMF)

Mathematical analysis of the theory delivers an equation which attempts to predict the binding energy of a nucleus in terms of the numbers of protons and neutrons it contains. There are five factors that contribute to the binding energy of nuclei. Let us now discuss them one by one.

• **Volume energy**:
When an assembly of spheres of the same size is packed together into the smallest volume, as we suppose is the case of nucleons within a nucleus, each interior sphere has 12 other spheres in contact with it. This term illustrates the idea that each nucleon only interacts with its nearest neighbors and binds to the nucleus at a specific binding energy. This is the dominant attractive term and will come with a +ve sign. Mathematical treatment is also very simple. Let's look at that. I have already told you the following

$$R = R_0\, A^{\frac{1}{3}}$$
$$\frac{4}{3}\pi\, R^3 = \frac{4}{3}\pi\, R_0^3\, A$$

Each nucleon has a binding energy which binds it to the nucleus. If U_v is binding energy per unit vol. of the nucleus then the total energy will be the following

$$\begin{aligned} E_V &= \frac{4}{3}\pi\, R_0^3\, A\, U_v \\ &= a_v\, A \end{aligned}$$

Therefore we get a term contributing to the energy proportional to A.

• **Surface energy**:
This term is a correction to the first term. The nucleons on the surface of the nucleus have fewer near neighbors, thus fewer interactions, than those in the interior of the nucleus. The effect is analogous to the surface tension of a liquid drop. Hence its binding energy is less. This surface energy takes that into account and is therefore negative. That is more is the surface area of a nucleus unstable the nucleus is going to be. Mathematical treatment is also very simple too here.

$$R = R_0\, A^{\frac{1}{3}}$$
$$4\,\pi\, R^2 = 4\,\pi\, R_0^2\, A^{\frac{2}{3}}$$

If U_s is binding energy per unit area of the nucleus then the total reduction in the energy will be the following

$$\begin{aligned} E_S &= -4\,\pi\, R_0^2\, A^{\frac{2}{3}} U_s \\ &= -a_s A^{\frac{2}{3}} \end{aligned}$$

Therefore we get a term proportional to $A^{\frac{2}{3}}$.

• **Coulomb energy**:
The Coulomb term represents the electrostatic repulsion between protons in a nucleus. The electric repulsion between each pair of protons in a nucleus also contributes toward decreasing its binding energy. The coulomb energy of a nucleus is equal to the work that must be done to bring together the protons from infinity into a spherical aggregate the size of the nucleus. The coulomb energy is negative because it arises from an effect that opposes nuclear stability. Mathematical treatment is also very simple but somewhat logical.
The potential energy between a pair of protons is given by

$$V = -\frac{e^2}{4\,\pi\,\epsilon\, R}$$

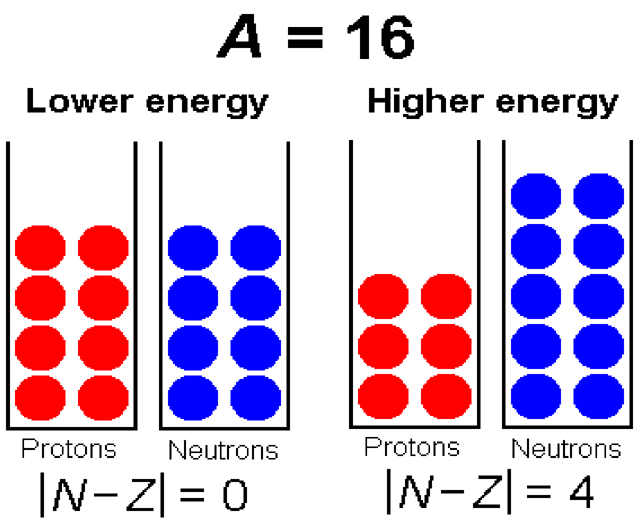
If there are Z numbers of protons in the nucleus then ${}^{Z}C_2$ numbers of pairs of protons will be there ie taking two protons together since for the repulsive force to develope there must be atleast two protons which is nothing but $\frac{Z(Z-1)}{2}$ numbers of pairs. Hence the coulomb energy will be the following

$$\begin{aligned} E_C &= -\frac{Z(Z-1)}{2}V \\ &= -\frac{Z(Z-1)}{2}\frac{e^2}{4\,\pi\,\epsilon\,R} \\ &= -\frac{Z(Z-1)e^2}{8\,\pi\,\epsilon\,R_0\,A^{\frac{1}{3}}} \\ &= -a_c\frac{Z(Z-1)}{A^{\frac{1}{3}}} \end{aligned}$$

Thus we get a term proportional to $\frac{Z(Z-1)}{A^{\frac{1}{3}}}$.

• **Asymmetry energy**:
This term is very ugly, I must say! Think about little bit of chemistry may be from your general course of even from higher secondary. Raoult's law! According to Raoults Law, in any two-component liquid with nonpolar attractive forces, the minimum in energy occurs when the two components occur in equal concentrations which will in turn generate a minimum in the vapor pressure and that will correspond to a maximum binding energy in the system. For nuclei with equal numbers of protons and neutrons, the nucleus is symmetric and it will be very stable. But what if the number of neutrons is greater than the number of protons. This energy associated as a correction in types of nuclei. This is a quantum effect arising from Pauli's exclusion principle which only allows two protons or two neutrons (with opposite spin direction) in each energy state. If a nucleus contains the same number of protons and neutrons then all the protons and neutrons will be filled up to the same maximum energy level. If, on the other hand, we exchange one of the neutrons by a proton then that proton would be guided by the exclusion principle to occupy a higher energy state, since all the below level states are already occupied. Well you can think of it this way too. Two different "pools" of states, one for protons and one for neutrons. Now, for example, if there are significantly more neutrons than protons in a nucleus, some of the neutrons will be higher in energy than the available states in the proton pool. If we could move some particles from the neutron pool to the proton pool, in other words change some neutrons into protons, we would significantly decrease the energy. The imbalance between the number of protons and neutrons causes the energy to be higher than it needs to be, for a given number of nucleons. This is the basis for the asymmetry term. Thus the asymmetry term accounts for the difference in the number of protons and neutrons in the nuclear matter.

Look at this figure. The left-side two U shaped structures is a nucleus and is a symetric one having equal nos. of protons and neutrons. Now what we want is that we should have a nucleus with the same mass number ie A. So that can be achieved by either changing protons into neutrons or vice versa which is in a way the nucleus is as if decaying via $\pm\beta$-decay. Now count the number of circles in the two U shape from the right. What have you got? Same value of A. But look at the positions of the neutrons, now they are occupying higher levels than before. Now compare with the original symteric one. you will see that the energy levels are quite different now leading to a different energy of the nucleus all together. Now the nucleus has also lost its symetry as it doesn't have same numbers of protons and neutrons even with the same value of A. Now the calculation of this energy is also somewhat simple.



If ϵ is energy per nucleonic level then the new neutron will occupy level higher in the energy by

$$\begin{aligned} \Delta\,E &= \text{number of new neutrons} \times \frac{\text{energy increased}}{\text{new neutrons}} \\ &= \frac{1}{2}(N-Z) \times \frac{1}{2}(N-Z)\frac{\epsilon}{2} \\ &= \frac{(N-Z)^2\,\epsilon}{8} \end{aligned}$$

Again it happens that greater is number of nucleons smaller will be the energy spacing hence ϵ will be inversely proportional to A. Hence

$$\begin{aligned}\Delta E &= \frac{(N-Z)^2}{8\,A} \\ &= -a_a\,\frac{(A-2Z)^2}{A}\end{aligned}$$

Thus we have a term proporsonal to $\frac{(A-2Z)^2}{A}$.

• **Pairing energy**:
Finally, there is one more ingredient to our binding energy recipe. The pairing energy it is called. This is again a correction term that arises from the tendency of proton pairs and neutron pairs to occur which actually occurs because of the different overlap of wavefunctions for pairs of nucleons in various states. In order to account for the binding energy, if number of protons and number of neutrons are both even the pairing energy is +ve, we subtract the same term if these are both odd, and do nothing if one is odd and the other is even. Experimentally it has been found that the pairing energy goes inversely as

$$E_p \propto \frac{1}{A^{\frac{3}{4}}}$$

Thus mathematically it can be written as

$$\begin{aligned}E_p &= \frac{a_p}{A^{\frac{3}{4}}} \qquad \text{even-even} \\ &= 0 \qquad \text{even-odd} \quad \text{or} \quad \text{odd-even} \\ &= -\frac{a_p}{A^{\frac{3}{4}}} \quad \text{odd-odd}\end{aligned}$$

Hence collecting all the energy term we get the Bethe-Weisacker's SEMF as

$$E_B = a_v\,A - a_s\,A^{\frac{2}{3}} - a_c\,\frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_a\,\frac{(A-2Z)^2}{A} \pm \frac{a_p}{A^{\frac{3}{4}}}, 0$$

Now it's time to get the values of the constants appearing before each of the indivisual terms. I urge you to remember these.
$a_v = 14.1$ MeV, $a_s = 13.0$ MeV, $a_c = 0.595$ MeV, $a_a = 19.0$ MeV, $a_p = 33.5$ MeV.

Q. The atomic mass of Zn isotope $_{30}Zn^{64}$ is 63.929 amu. Compare the binding energy from nuclear composition and predicted by SEMF?
Answer: So Zn has 30 protons and 34 neutrons. Therefore using the equation

$$\begin{aligned}E &= [(n_{prot}m_{prot} + n_{neut}m_{neut}) - M] \times 931.5 \quad MeV \\ &= [(30 \times 1.0072 + 34 \times 1.0086) - 63.929] \times 931.5 \quad MeV \\ &= 559.1 \quad MeV\end{aligned}$$

Now using SEMF, well we will only replace the mass numbers (As) by 64 and the atomic numbers (Zs) by 30 and replace the constants by their repective values, we get

$$\begin{aligned}E &= a_v\,A - a_s\,A^{\frac{2}{3}} - a_c\,\frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_a\,\frac{(A-2Z)^2}{A} + \frac{a_p}{A^{\frac{3}{4}}} \quad MeV \\ &= 14.1 \times 64 - 13.0 \times 64^{\frac{2}{3}} - 0.595 \times \frac{30(30-1)}{64^{\frac{1}{3}}} - 19.0 \times \frac{(64-2\times 30)^2}{64} + \frac{33.5}{64^{\frac{3}{4}}} \quad \text{MeV} \\ &= 561.7 \quad \text{MeV} \qquad\qquad \text{(do your calculations)}\end{aligned}$$

Now the difference between these two is less than 0.5%. And the plus sign in the pairing energy is because $_{30}Zn^{64}$ is an even-even nucleus.

Q. Derive a formula for the atomic number of the most stable isotope isobar of a given A and use it to find the most stable isobar of A = 25
Answer: *Well this we will handle using differentials. We know that for maxima or minima we do the first derivative find the value of x, then we do the second derivative put the value of x and see whether it is coming positive or negative. If negative it is maxima else minima.*
Thus for maximum stability we must have

$$\frac{dE}{dZ} = 0$$

Dr. Upakul Mahanta, Department of Physics, Bhattadev University, Bajali

Hence following this we have and we will take partial instead of total (well it doesn't make any difference)

$$\begin{aligned}
\frac{\delta E}{\delta Z} &= \frac{\delta}{\delta Z}\left[a_v\, A - a_s\, A^{\frac{2}{3}} - a_c\, \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_a\, \frac{(A-2Z)^2}{A} \pm \frac{a_p}{A^{\frac{3}{4}}}, 0\right] \\
&= 0 + 0 - \frac{a_c}{A^{\frac{1}{3}}}(2Z-1) + \frac{4a_a}{A}(A-2Z) + 0
\end{aligned}$$

Thus

$$-\frac{a_c}{A^{\frac{1}{3}}}(2Z-1) + \frac{4a_a}{A}(A-2Z) = 0$$

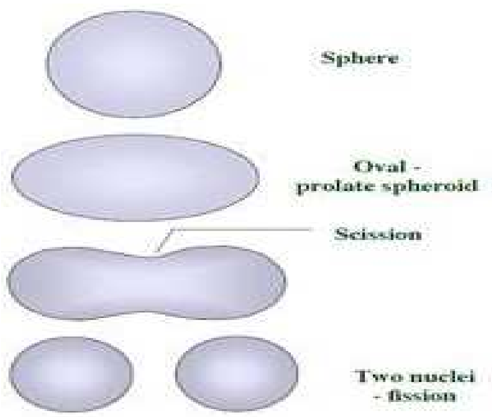
$$\begin{aligned}
Z &= \frac{a_c A^{\frac{-1}{3}} \; + 4a_a}{2a_c A^{\frac{-1}{3}} + 8a_a A^{-1}} \\
&= \frac{0.595 A^{\frac{-1}{3}} \; + 76}{1.19 A^{\frac{-1}{3}} + 152 A^{-1}} \\
&= \frac{0.595 \times 25^{\frac{-1}{3}} \; + 76}{1.19 \times 25^{\frac{-1}{3}} + 152 \times 25^{-1}} \\
&\approx 12
\end{aligned}$$

Thus we can conclude that a nucleus with Z= 12 and A= 25 is going to be most stable amongst the isobars.

Q. Predict which one is the most stable nucleus amongst ${}_8O^{16}$,${}_8O^{17}$ and ${}_8O^{18}$ (typical for your exam)
Answer: *Similar treatment like the above. But in this case you have differentiate w.r.to A. See A is the variable in the isotopic family. Then find the value of A. And see which value does this A close to. Say if you get A=* 15.8 *then it is close to* 16 *or even say* 16.4 *then it is also* 16. 17 *is too far right. Solve it. Good luck.*

- **Explanation of Nuclear Fission on the basis of Liquid Drop Model**

By this time we have come to know that the atomic nucleus behaves like the molecules in a drop of liquid. But in this nuclear scale, the fluid is made of nucleons (protons and neutrons), which are held together by the strong nuclear force. The interior nucleons are completely surrounded by other attracting nucleons just like molecules did in case of a liquid drop. In the ground state the nucleus is spherical. If the sufficient kinetic or binding energy is added, this spherical nucleus may be distorted into a dumbbell shape and due to positive charge repulsion on the two ends splits into two fragments hence, forming daughter two nuclei. This process is what we call nuclear fission.



- **Achievements of this model:**

1. It predicts the atomic masses and binding energies of various nuclei to a larger accuracy.
2. It predicts emission of alpha and beta particles in radioactivity.
3. The theory of compound nucleus, which is based on this model, explains the basic features of the nuclear fission process.

- **Failures of this model:**

1. It fails to explain the extra stability of certain nuclei, with the numbers of protons or neutrons are 2, 8, 20, 28, 50, 82 or 126 etc
2. It fails to explain the measured magnetic moments of many nuclei.
3. It also fails to explain the spin and parity (explained later on) of nuclei.
4. It is also not successful in explaining the excited states in most of the nuclei.

3.3 Shell model of the nucleus

The basic assumption of the liquid drop model is that each nucleon interacts only with its nearest neighbour. Though it explains nuclear fission, sphericity of the nucleus and binding energy of the nuclei to a large extent but few significant

things it fails to explain. Which are
• There are some peaks or kinks the in binding energy/nucleon curve (see fig. 1.1)
• It underestimate the actual binding energies of some **magic nuclei** for which either the number of neutrons N = (A - Z) or the number of protons, Z is equal to one of the **magic numbers** (a fancy term used by the nuclear physicist) which are $2, 8, 20, 28, 50, 82$ etc. These numbers are exceptional in the sense that any nucleus which posseses any of these values in terms of neutrons or protons or sum of these two are highly stable nuclei. For example for $_{28}Ni^{56}$ the Liquid Drop Model predicts a binding energy of 477.7 MeV, whereas the measured value is 484.0 MeV. Likewise for $_{50}Sn^{132}$ the Liquid Drop model predicts a binding energy of 1084 MeV, whereas the measured value is 1110 MeV. You know that an α-particle is exceptionally stable because its proton number and neutron number are both equal to 2, a magic number. An α-particle is therefore said to be doubly magic because they contain filled shells of both protons and neutrons.
• Changes in separation energies (the energy required to remove the last neutron (or proton)) for certain numbers of neutron and protons.
• If N is magic number then the cross-section for neutron absorption is much lower than for other nuclides.

The shell model is an attempt to solve these ambiguities which a model of the nucleus that uses the Pauli exclusion principle to describe the structure of the nucleus in terms of energy levels. The shell model is partly analogous to the atomic shell model which describes the arrangement of electrons in an atom, in that a filled shell results in greater stability. In the Shell Model it is assumed that each nucleon in the nucleus moves in a net attractive potential that represents the avg. effect of its interaction. The potential has a constant depth inside the nucleus and outside the nucleus it goes to zero within a distance equal to the range of the nuclear force. It almost like a 3D potential with round edges. And in the ground state of the nucleus the nucleons are filled without violating the Pauli's exclusion principle. And that immediately excludes the possibility of nucleon-nucleon collision. But two nucleons can exchange their quantum states which will be indistinguishable. Hence all the nucleons that constitute the nucleus can move freely inside the ground state nucleus. So, this model is also called as independent particle model. And the behaviour of each nucleon can be understood by solving the Schrodinger equation for that potential. This Shell Model plays the same role in nuclear physics as Hartree-Fock theory in atomic physics. However there are some similarities and differences between these two. Let's have a look at it.

Table 3.1: Similarities between Shell Model and Hartree-Fock Theory

Shell Model	**Hartree-Fock Theory**
1. Nucleons move in an attractive potential	1. Electorns move in an attractive potential
2. Nucleons obey Pauli's exclusion principle	2. Electrons obey Pauli's exclusion principle
3. Nuclear potential V(r) depends on n and l	3. Atomic potential V(r) depends on n and l
4. Nuclear spin-orbit interaction is present	4. Atomic spin-orbit interaction is present

Table 3.2: Differences between Shell Model and Hartree-Fock Theory

Shell Model	**Hartree-Fock Theory**
1. Potential V(r) is square well with round edge	1. Potential V(r) is spherically symmetric
2. n = radial node quantum number	2. n = principal quantum number
($n_{principal} = n_{radial} + l$)	
3. No upper limit for l	3. There is an upper limit for l
4. Strong inverted spin-orbit interaction (S • L)	4. Spin-orbit interaction (L • S)
$(S \bullet L)_{nuclear} = 20 \times (L \bullet S)_{atomic}$	
5. Spin-orbit interaction is not magnetic in origin	5. Spin-orbit interaction is magnetic in origin

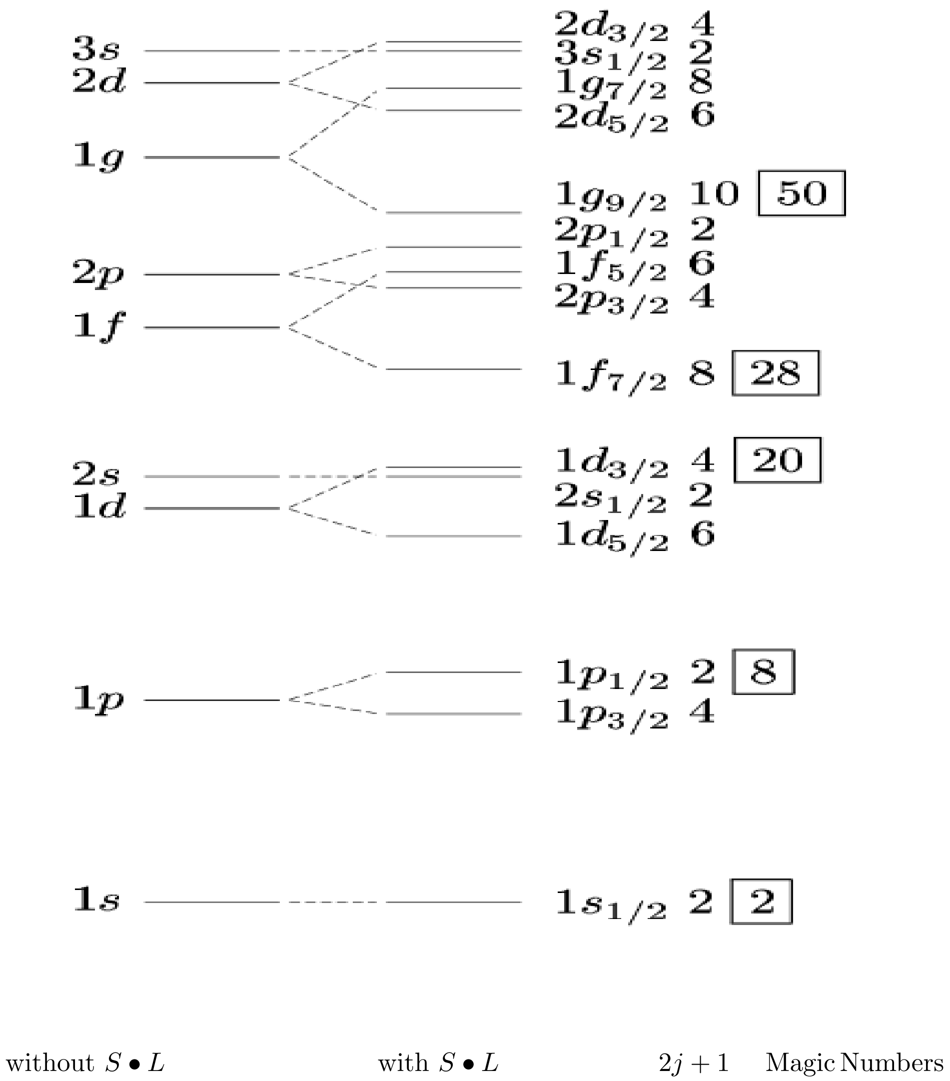
Keeping these things in mind if one proceeds to fill up the various states then in case of atomic physics you have **Aufbau Principle as e^-s have to be filled in various orbitals as increasing order of their energies.** Right! Here in the case of nucleus I can't give you an Aufbau Principle. But I can give you a mnemonic which in German as follows

spuds if pug dish of pig

Its means "eat potatos if the pork is bad ". But we have nothing to do with eating. But here is what you should remember. **Delete all the vowels except the last one** in the above sentence or what. If you do that you will be left with the following

s p d s f p g d s h f p i g

Yes exactly. These are my orbitals where I am going to put my nucleons. Let's then do the final job. The nuclear energy levels going to be filled up by the nucleons in various shells.



• **<u>The construction of this figure:</u>**

Well how have I contructed this or what are the key features? Following are the points that you should remember.

• Without S • L coupling there would not have been any splitting in the energies. Thus the first vertical contruction.

• But since there has been S • L coupling the energy levels gets splitted out. Now the splitting is similar to the atomic case. It depends on values of azimuthal quantum number l and spin quantum number s. For instance for s orbital l value is 0. Hence no splitting. Then for p orbital l value is 1 and s value is $\pm\frac{1}{2}$. Thus the total angular momentum, j will be $|l+s|= \frac{3}{2}$ and $|l-s|= \frac{1}{2}$. Now see the energy level diagram. You will see the term symbol like $1p_{\frac{1}{2}}$ lying up and $1p_{\frac{3}{2}}$ lying down which is exactly opposite to the case of atomics. In atomic case it would have been $1p_{\frac{1}{2}}$ lying down and $1p_{\frac{3}{2}}$ lying up. Can you imagine why this is so? Well here we have S • L coupling which is opposite to L • S coupling in atomic physics. Similar arguments for d, f orbitals etc too.

• Now number of nucleons to be put in each orbital will be equal to $2j+1$ similar to atomic physics. Thus in s orbital $2\times\frac{1}{2}+1 = 2$ numbers of nucleon can be placed. In $1p_{\frac{3}{2}}$, $2\times\frac{3}{2}+1 = 4$ numbers of nucleon can occupy that orbital. And likewise.

• Here is the catch of giving the digits before the s, p, d etc orbitals. Whenever the first appearence of the orbitals come it is given 1 second appearences are given 2 and similarly others.

• This diagram has to be filled up by both neutrons and protons independently.

• The gap in the energy levels are subsequently decreasing in the vertical direction. Did you see that.

• **Prediction of Nuclear Spin on the basis of Shell Model**

As I have said that both neutrons and protons have to be filled independently that means you need to have two such figures. Now from the nucleus find out how many number of neutrons and protons are there. Once that is done keep on filling all the neutrons and protons as according to the $2j+1$ values in various orbitals of course obeying Pauli's exclusion principle. ie say in $1s_{\frac{1}{2}}$ orbital one up and one down, in $1p_{\frac{1}{2}}$ orbital one up and one down and in $1p_{\frac{3}{2}}$ orbital one up, one down, one up the last one down, a total of 4 protons and 4 neutrons and similarly for others as well. Once all the nucleons are used up then go to top level, the last occupied state and look how they are arranged. If there is any unpaired nucleon remaining, the spin of the nucleus will be the value of the angular momentum of that state. And if cases like one proton and one neutron is remaining unpaired than spin of the nucleus will be the algebraic sum of the angular momentums of those two states. I will show you in the class. It's very easy to understand. But remember by default we will always take the up spin as +ve values and down spin as -ve values and also we will always fill the up spin first.

• **Prediction of Nuclear parity on the basis of Shell Model**
You might not have heard about parity. But it's a day to day business though not we perceive it mathematically. When you comb your hair in front the looking glass and raise your right hand the mirror image will raise the left hand if turn your back. That means you and your image share odd parity. Mathematically speaking, the behaviour of the wave function under the reflection of space coordinates through the origin determines the parity of the system. Under reflection of space coordinates, the wave function may change sign or may remain unchanged. In case the wave function does not change sign upon reflection of space coordinates, its parity is even and if it changes sign its parity is odd. ie $\psi(-r) = \psi(r)$ than the parity is even and if $\psi(-r) = -\psi(r)$ than the parity is odd. That is it is mere coming of a minus sign.
To find the parity of the nucleus we also, here, keep on filling all the neutrons and protons, separately, as according to the $2j + 1$ values in various orbitals obeying Pauli's exclusion principle. And once that is done go to top level and count how many protons and neutrons are occupied that level along with the value of the azimuthal quantum number, l for that occupied orbital. From atomic physics you know that for s orbital the value of l is 0, for p orbital it is 1, for d orbital it is 2 and likewise for the others. So finally the parity, P of the nucleus will be

$$P = (-1)^{\sum l_p + \sum l_n}$$

where $\sum l_p$ is the total l value with the contribution coming from all the last level occuping protons. For instance in 3 protons stands last in the $1p_{\frac{3}{2}}$ orbital then $\sum l_p$ will be $1 + 1 + 1 = 3$. If 2 protons occupies $1f_{\frac{7}{2}}$ then $\sum l_p$ will be $3 + 3 = 6$. Similarly for the neutrons too, ie the $\sum l_n$. And then finally we will raise -1 to the powered sum of $\sum l_p$ and $\sum l_n$.

Q. Predict the spin and parity of the following nuclei.

$$_6C^{12}, \quad _6C^{13}, \quad _7N^{14}, \quad _7N^{15}, \quad _8O^{16}, \quad _8O^{17}, \quad _8O^{18}, \quad _9F^{19}, \quad _{10}Ne^{21} \text{ etc}$$

• **Achievements of this model:**
1. It Expalins nuclear spin and parity.
2. It also explains magnetic moments for lighter nuclei.
3. It also speaks about excited states of nuclei.

• **Failures of this model:**
1. For heavier nuclei it fails to predict magnetic dipole moments or the spectra of excited states very well.
2. The Shell Model also does not give predictions of electric quadrupole moments of the nuclei.

3.4 Collective Model of the nucleus

3.4.1 Introduction

Collective model, also called *unified model*, description of atomic nuclei that incorporates aspects of both the shell nuclear model and the liquid-drop model to explain certain magnetic and electric properties that neither of the two separately can explain. In the liquid-drop model, nuclear structure and behaviour are explained on the basis of statistical contributions of all the nucleons. In the shell model, nuclear energy levels are calculated on the basis of a single nucleon (proton or neutron) moving in a potential field produced by all the other nucleons. Nuclear structure and behaviour are then explained by considering single nucleons beyond a passive nuclear core composed of paired protons and paired neutrons that fill groups of energy levels, or shells.
However some difficulties have also crept in while
• Explaining the nuclear spectra for nuclei without magic numbers.
• With increase in particle number, the feasibility to carrying out shell model calculations.
Moreever, while moving further away from the closed shells, some simple and systematic features start to show up. For example, odd-A nuclei in the midshell regions are characterized by exceptionally large positive quadrupole moments Q and even-even nuclei in the same region have a rather low-lying first excited state with $J = 2^+$ which can be explained by a simple formula like

$$E(J) = A\,J(J+1) + B\,J^2(J+1)^2$$

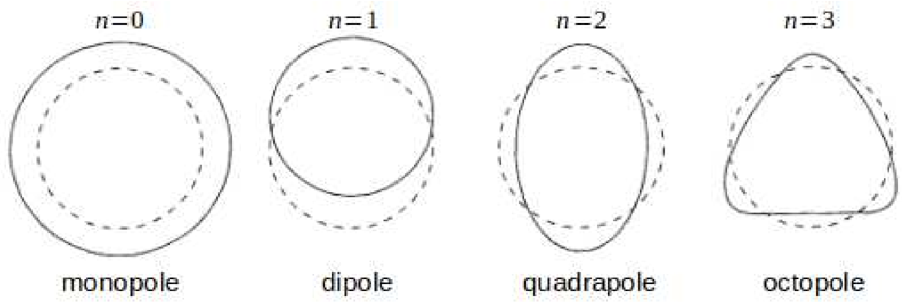
Thus in many nuclei ground states or low-energy excitations (E) involve a coordinated, large-amplitude motion of many nucleons. Nuclear properties which are determined by such a coordinated, large-amplitude motion of many nucleons are often referred to as collective properties which are often quite simple to describe in terms of deformation of nuclear surface. This leads to the development of the model, forwarded by A. Bohr and B. Motelsson.

Assumptions made:
• The nuclear core is thought of as a liquid drop with constant density.
• The dynamics of the nuclear surface is described by surface parameters due to which the surface circulates and a stable tidal bulge directed toward the rotating unpaired nucleons outside the bulge gets formed.
The tide of positively charged protons constitutes a current that in turn contributes to the magnetic properties of the nucleus. The increase in nuclear deformation that occurs with the increase in the number of unpaired nucleons accounts for the measured electric quadrupole moment, which may be considered a measure of how much the distribution of electric charge in the nucleus departs from spherical symmetry. Thus in the collective model, high-energy states of the nucleus and certain magnetic and electric properties are explained by the motion of the nucleons outside the closed shells (full energy levels) combined with the motion of the paired nucleons in the core.

Let us first consider a spherical shell defined by a constant radius given by $R(\theta, \phi) = R_0$ =constant. Now deforming the shell by changing the radius slightly can be a complicated task since now the radius becomes a function of the polar and azimuthal angles θ and ϕ. But using spherical harmonics (*Recall Legendre polynomial*) having rank n and m represented as $Y_n^m(\theta, \phi)$ this can be done. Thus sphericity is lost by

$$R(\theta,\, \phi\, t) \;=\; R_0\, \left[1 + \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \alpha_{n,\,m}(t)\, Y_n^m(\theta,\, \phi)\right] \tag{3.1}$$

where $R(\theta,\, \phi\, t)$ denotes the nuclear radius in the direction $(\theta,\, \phi)$ at time t. The multipole terms in the above expansion have different physical meaning and the shape they describe.
• **For** $n = 0$ **:** It is called as monopole mode and describes the compression of nuclear matter. The incompresibility of the nuclear matter which implies volume conservation. For low energy spectra this mode in unimportant since it requires a large amount of energy to compress nuclear matter.
• **For** $n = 1$ **:** is called as the dipole mode and describes the shift in the center of mass of the nucleus and does not refer to any physical change. But the nuclear deformation should not change the position of the centre of mass of a nucleus. Hence this mode is also less important.
• **For** $n = 2$ **:** is called as the quadrupole deformation mode. It describes the deviation of a nuclear surface from the spherical surface and is the most important term in nuclear spectroscopy of bound states. The surface looks like an ellipsoid.
• **For** $n = 3$ **:** is called as the octupole deformation mode. Like quadrupole mode it also describes the deviation of nuclear surface from the spherical surface. In this case the surface is like a pear.



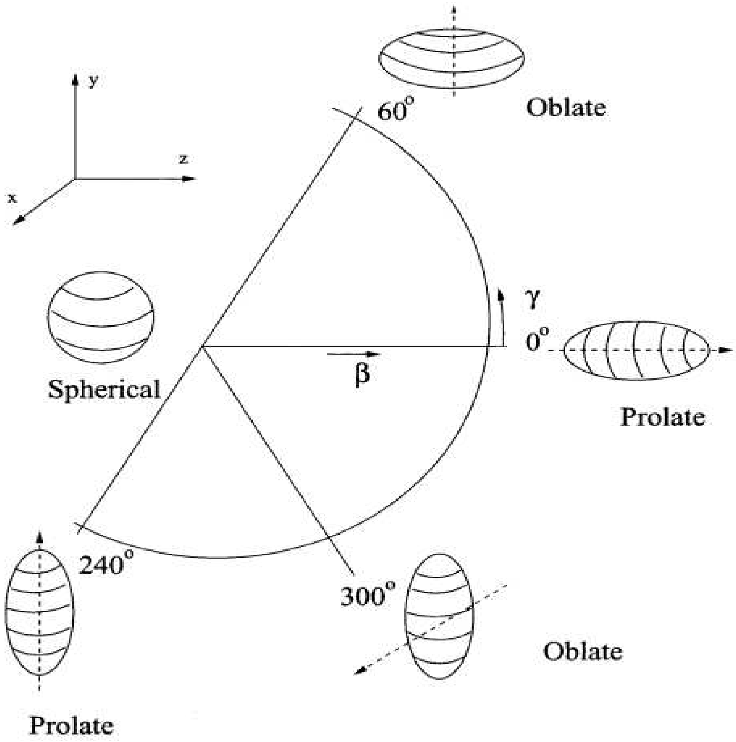
3.4.2 Quadrupole deformation parameters: Bohr-Wheeler parametrization

For simplicity, let us concentrate on the first independent deformation which is of rank $n = 2$ (quadrupole) and how many parameters are needed to describe static quadrupole deformation. As the nuclear surface is real, for $n = 2$ and making transformation to intrinsic frame, there will be five coefficients for the deformation parameter $\alpha_{n,\,m}$ namely $\alpha_{2,\,-2}$, $\alpha_{2,\,-1}$, $\alpha_{2,\,0}$, $\alpha_{2,\,1}$ and $\alpha_{2,\,2}$ reduces to real parameters out of which there will be only two which will be independent. *(I am sorry to say that you really have to remember these things. Calculations involved here are realy for nuclear specialists).* These are $\alpha_{2,\,0}$ and $\alpha_{2,\,2} = \alpha_{2,\,-2}$. The two parameters of static quadrupole deformation in the intrinsic system are often chosen as the Bohr-Wheeler parameters β and γ. So the nuclear surface in the intrinsic frame can be written as

$$\begin{aligned}
R(\theta,\, \phi,\, t) \;&=\; R_0\, \left[1 \,+\, \alpha_{2,\,0} Y_2^0(\theta,\, \phi) \,+\, \alpha_{2,\,2} Y_2^2(\theta,\, \phi) \,+\, \alpha_{2,\,-2} Y_2^{-2}(\theta,\, \phi)\right] \\
&=\; R_0\, \left[1 \,+\, \beta\cos\gamma\, \sqrt{\frac{5}{16\,\pi}}(3\cos^2\theta \,-\, 1) \,+\, \frac{1}{\sqrt{2}}\,\beta\sin\gamma\, \sqrt{\frac{15}{32\,\pi}}\, e^{i\,2\,\phi}\sin^2\theta \,+\, \frac{1}{\sqrt{2}}\,\beta\sin\gamma\, \sqrt{\frac{15}{32\,\pi}}\, e^{-i\,2\,\phi}\sin^2\theta\right] \\
&=\; R_0\, \left[1 \,+\, \beta\cos\gamma\, \sqrt{\frac{5}{16\,\pi}}(3\cos^2\theta \,-\, 1) \,+\, 2\,\frac{1}{\sqrt{2}}\,\beta\sin\gamma\, \sqrt{\frac{15}{32\,\pi}}\, \cos{(2\,\phi)}\sin^2\theta\right] \qquad \text{(considering the real part)}
\end{aligned}$$

$$R(\theta,\,\phi,\,t)\;=\;R_0\left[1\,+\,\beta\sqrt{\frac{5}{16\,\pi}}\left(\cos\gamma\,(3\cos^2\theta\;-\;1)\;+\sin\gamma\sqrt{\frac{3}{\pi}}\cos\left(2\,\phi\right)\sin^2\theta\right)\right]$$

From the above expression the change of the length of principal axis from the spherical radius R_0 in the intrinsic frame can be obtained by putting different values of θ and ϕ. From these expressions, the shape of the nucleus for different values of deformation pa- rameter can be easily visualized. The parameter β describes the degree of deformation while the parameter γ describes the shape of the nucleus.



Chapter 4

Nuclear Reactions

Dr. Upakul Mahanta, Department of Physics, Bhattadev University, Bajali

4.1 Introduction

The study of nuclear reactions is important for a number of reasons. For example, life on earth would not be possible without the energy provided to us by the sun. That energy is the energy released in the nuclear reactions that drive the sun and other stars. For better or worse, the nuclear reactions, fission and fusion, are the basis for nuclear weapons, which have shaped much of the geopolitical dialog for the last 50 years. Apart from the intrinsically interesting nature of these dynamic processes, their practical importance would be enough to justify their study. Nuclear reactions and nuclear scattering are used to measure the properties of nuclei. Reactions that exchange energy or nucleons can be used to measure the energies of binding and excitation, quantum numbers of energy levels, and transition rates between levels. In order for a nuclear reaction to occur, the nucleons in the incident particle, or projectile, must interact with the nucleons in the target. Thus the energy must be high enough to overcome the natural electromagnetic repulsion between the protons. This energy "barrier" is called the **Coulomb barrier**. If the energy is below the barrier, the nuclei will bounce off each other. So atypical nuclear reaction can be written as

$$\text{Target nucleus X + projectile a} \rightarrow \text{Product nucleus Y + ejectile b}$$

4.2 Various types of Nuclear reactions

• **Elastic scattering**
It occurs, when no energy is transferred between the target nucleus and the incident particle. for example

$$Pb^{208}(n,n)Pb^{208}$$

• **Inelastic scattering**
It occurs, when energy is transferred to the product. The difference of kinetic energies is saved in excited nucleus.

$$Ca^{40}(\alpha,\alpha')Ca^{40*}$$

• **Capture reactions**
Both charged and neutral particles can be captured by nuclei. This is accompanied by the emission of γ-rays. Neutron capture reaction produces radioactive nuclides.

$$U^{238}(n,\gamma)U^{239}$$

• **Transfer Reactions**
The absorption of a particle accompanied by the emission of one or more particles is called the transfer reaction.

$$He^4(\alpha,p)Li^7$$

• **Fission reactions**
Nuclear fission is a nuclear reaction in which the nucleus of an atom splits into smaller parts (lighter nuclei). The fission process often produces free neutrons and photons (in the form of gamma rays), and releases a large amount of energy.

$$U^{235}(n,3\,n)\,\text{fission}\quad\text{products}$$

• **Fusion reactions**
It Occurs when, two or more atomic nuclei collide at a very high speed and join to form a new type of atomic nucleus.

$$H^3(d,n)\,He^4$$

• **Spallation reactions**
It occurs, when a nucleus is hit by a particle with sufficient energy and momentum to knock out several small fragments or, smash it into many fragments.

In order to get this reaction some conservation laws have to followed for sure. These are just below
• **Conservation of Mass Number:**
It demands the sum of reactants' mass number must be same as sum of products' mass number.

$$\sum A_{reac} = \sum A_{prod}$$

• **Conservation of Atomic Number:**
It demands the sum of reactants' total charge must be same as sum of products' total charge.

$$\sum Z_{reac} = \sum Z_{prod}$$

• **Conservation of Energy:**
It demands the total energy of reactants' must be equal to the total energy of the products. By total energy I mean the sum of rest mass energy and the kinetic energy of the nuclei. Or equally you can say the total relativistic energy always remains the same.

$$\sum E_{reac}(\text{rest+kinectic}) = \sum E_{prod}(\text{rest+kinectic})$$

• **Conservation of Linear Momemtum:**
It demands the total vector sum of reactants' linear momentum must also remain the same as that of the products'.

$$\sum \vec{p}_{reac} = \sum \vec{p}_{prod}$$

• **Conservation of Angular Momemtum:**
It demands the total vector sum of reactants' angular momentum must also remain the same as that of the products'. Here you all know that angular momentum is nothing but the spin of the involved nuclei in the reaction. (The spins will be calculated from Shell Model)

$$\sum \vec{L}_{reac} = \sum \vec{L}_{prod}$$

• **Conservation of Isospin:**
It demands the total vector sum of reactants' angular momentum must also remain the same as that of the products'.

$$\sum \vec{T}_{reac} = \sum \vec{T}_{prod}$$

4.3 Q-value of a nuclear reaction

In nuclear physics, the Q value for a reaction is the amount of energy released or absorbed by that reaction. So basically it is the energy balance term in a nuclear reaction. The energy conservation relation, enables the general definition of Q based on mass-energy equivalence. To calculate the Q-value look at the following calculation

$$X + a \rightarrow Y + b$$

So conservation of demands

$$TE_X + TE_a = TE_Y + TE_b$$
$$(\text{kinetic} + \text{rest})_X + (\text{kinetic} + \text{rest})_a = (\text{kinetic} + \text{rest})_Y) + (\text{kinetic} + \text{rest})_b$$
$$(KE_X + M_{0X}c^2) + (KE_a + M_{0a}c^2) = (KE_Y + M_{0Y}c^2) + (KE_b + M_{0b}c^2)$$

Here KE stands for kinetic and *rest* stands for rest mass energy respectively and M, m s are masses of the involved particles. Now if you take the target nucleus to be at rest then $KE_X = 0$. In that case the last expression becomes

$$M_{0X}c^2 + (KE_a + m_{0a}c^2) = (KE_Y + M_{0Y}c^2) + (KE_b + m_{0b}c^2)$$

Now the Q-value is

$$Q = TE_{final} - TE_{initial} = (E_Y + E_b) - E_a$$
$$= [(M_{0X} + m_{0a}) - (M_{0Y} + m_{0b})] \times c^2 = \Delta m \times 931.5\,\text{MeV}$$

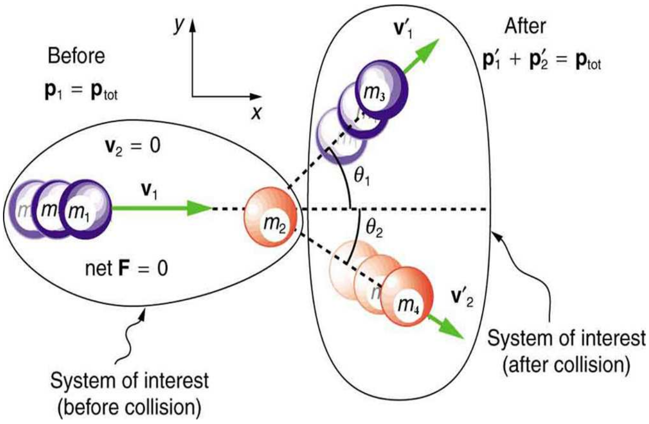
This Δm will be coming in *amu* s. So multiplying with 931.5 will give you the energy released.
• **Cases for Q-value**
1. $(M_{0X} + m_{0a}) > (M_{0Y} + m_{0b})$ will imply $Q > 0$, is termed as *exoenergic reaction*
2. $(M_{0X} + m_{0a}) < (M_{0Y} + m_{0b})$ will imply $Q < 0$, is termed as *endoenergic reaction*
3. $(M_{0X} + m_{0a}) = (M_{0Y} + m_{0b})$ will imply $Q = 0$, is termed as *elastic reaction*

So, in principle you can calculate the Q-value of the nuclear reaction if the masses are given. Thus the final definition **Q-value of the reaction is defined as the difference between the sum of the masses of the initial reactants and the sum of the masses of the final products, in energy units.**

4.4 Nuclear reaction kinematics

What if you don't know the mass of target nuclues. In general the particle with whom you are going to bombard the target nucleus is generally known. And whatever are particles going to get produced, you have the desire to know the masses of them. So in principle you are going to have three known masses and an unknown. So can it be a way to determine the Q-value in such cases. And the answer is YES. But what we are going to compensate for that is the introduction of a scattering angle in the equation and this is going to be performed in laboratory frame of course. Let's now look at the calculations for that.

Consider a reaction in which the bombarding particle strikes a target a rest. After the collision the product nucleus goes in a direction θ_2 and the ejectile in a direction θ_1 w. r. to the x-axis ie with the incoming direction of the projectile. In the fig.



$m_1 = m_a =$ mass of the projectile

$m_2 = m_X =$ mass of the target nucleus

$m_3 = m_Y =$ mass of the ejectile

$m_4 = m_b =$ mass of the product nucleus

Now conservation of momentum demands that x-axis momentum and y-axis momentum have to equal before and after collision. Since momentum is a vector quantity therefore you have treat it vectorially. Thus the x-axis momentum is

$$\begin{aligned} \vec{p}_x(\text{ before collision}) &= \vec{p}_x(\text{after collision}) \\ \vec{p}_a &= \vec{p}_Y\, cos\theta_2 + \vec{p}_b\, cos\theta_1 \\ \vec{p}_Y\, cos\theta_2 &= -(\vec{p}_b\, cos\theta_1 - \vec{p}_a) \end{aligned} \tag{4.1}$$

Similarly the y-axis momentum is

$$\begin{aligned} \vec{p}_y(\text{before collision}) &= \vec{p}_y(\text{after collision}) \\ 0 &= \vec{p}_Y\, sin\theta_2 - \vec{p}_b\, sin\theta_1 \\ \vec{p}_Y\, sin\theta_2 &= \vec{p}_b\, sin\theta_1 \end{aligned} \tag{4.2}$$

Now squaring and adding the last two numbered equation we get

$$\begin{aligned} p_Y^2\, cos^2\theta_2 + p_Y^2\, sin^2\theta_2 &= [-(p_b\, cos\theta_1 - p_a)]^2 + p_b^2\, sin^2\theta_1 \\ p_Y^2[cos^2\theta_2 + sin^2\theta_2] &= p_b^2\, cos^2\theta_1 - 2\, p_b\, cos\theta_1\, p_a + p_a^2 + p_b^2\, sin^2\theta_1 \\ p_Y^2 &= p_b^2 + p_a^2 - 2\, p_a\, p_b\, cos\theta_1 \\ 2\, M_Y\, E_Y &= 2\, m_b\, E_b + 2\, m_a\, E_a - 2\sqrt{2\, m_b\, E_b\, 2\, m_a\, E_a}\, cos\theta_1 \qquad \text{since} \quad \frac{p^2}{2m} = E \\ M_Y\, E_Y &= m_b\, E_b + m_a\, E_a - 2\sqrt{m_b\, E_b\, m_a\, E_a}\, cos\theta_1 \\ E_Y &= \frac{m_b}{M_Y}\, E_b + \frac{m_a}{M_Y}\, E_a - \frac{2}{M_Y}\sqrt{m_a\, m_b\, E_a\, E_b}\, cos\theta_1 \end{aligned}$$

From the last section we have known that

$$Q = (E_Y + E_b) - E_a$$

Now substituting the value of E$_Y$ in the last equation we get

$$\begin{aligned} Q = (E_Y + E_b) - E_a &= \left(\frac{m_b}{M_Y}\, E_b + \frac{m_a}{M_Y}\, E_a - \frac{2}{M_Y}\sqrt{m_a\, m_b\, E_a\, E_b}\, cos\theta_1 + E_b\right) - E_a \\ &= \left[1 - \frac{m_a}{M_Y}\right] E_a + \left[1 + \frac{m_b}{M_Y}\right] E_b - \frac{2}{M_Y}\sqrt{m_a\, m_b\, E_a\, E_b}\, cos\theta_1 \end{aligned}$$

Thus all you have to do is to measure the scattering angle and the masses of the projectile and the product nucleus to get to the Q-value.

4.5 Bohr's compound nuclear theory

It's a description of atomic nuclei proposed (1936) by the Danish physicist Niels Bohr to explain nuclear reactions as a two-stage process comprising the formation of a relatively long-lived intermediate nucleus and its subsequent decay.

However the compound nucleus absolutely forgets about its past while it gets formed. Few properties are defined in case of compound nuclear theory

• A bombarding particle loses all its energy to the target nucleus and becomes an integral part of a new, highly excited, unstable nucleus, called a compound nucleus.

• The formation stage takes a period of time approximately equal to the time interval for the bombarding particle to travel across the diameter of the target nucleus (about 10^{-21} second).

• After a relatively long period of time (typically from 10^{-19} to 10^{-15} second) and independent of the properties of the reactants, the compound nucleus disintegrates, usually into an ejected small particle and a product nucleus. Symbolically it can be written as the following

$$X + a \rightarrow [C]^{\star} \rightarrow Y + b$$

where $[Z]^*$ is the compound nucleus. The formation of the compound nucleus will guarantee the production of other nuclei. But what will form that will be governed by the conservation rules discussed earlier. These have to be keep in mind always.

4.6 Ghosal's Experiment

4.6.1 Aim

It was conducted by S.N. Ghosal to verify Bohr's compound nuclear theory.

4.6.2 The theory behind the experiment

According to Bohr's compound nuclear theory, as discussed in the earlier section a nuclear reaction proceeds in two stages: first, the formation of a quasistable high energetic compound nucleus through the absorption of the incident particle by the target nucleus; second, the disintegration of the compound nucleus by the emission of either the original incident particle (scattering) or the emission of another particle with a different nucleus or leaving the same nucleus with an emission of a photon. The equation of compund nuclear theory as written earlier is

$$X + a \rightarrow [C]^{\star} \rightarrow Y + b$$

This permits us to express the cross section of a reaction

$$\sigma(a, b) = \sigma_a(E)\, P_b(E')$$

where $\sigma_a(E)$ is the cross section for the absorption of the particle a of kinetic energy E by the target nucleus to form the compound state $[C]^\star$. $P_b(E')$ is the probability of disintegration of $[C]^\star$ into the final state $Y + b$. Now $E' = E + \mathrm{BE}_a$ is the excitation energy of the compound state $[C]^\star$, being the binding energy of the particle a to the target nucleus A.

If the compound nucleus $[C]^\star$ is now formed in the same state of excitation by another process $X' + a'$, the cross section for disintegration into the same final state, $Y + b$, will be given by

$$\sigma(a', b) = \sigma_{a'}(E')\, P_b(E')$$

where E' is the kinetic energy of the incident particle a'. Because of the difFerences in the binding energies between the two cases E' will be difFerent from the kinetic energy E of a of the previous case. $P_b(E')$ will be the same in the two cases, because of the basic assumption that the mode of decay of the compound nucleus $[C]^\star$ is independent of the mode of its formation.

If $[C]^\star$ decays into a difFerent final state, say $Z + d$, the corresponding cross sections for the reaction will be given by

$$\sigma(a, d) = \sigma_a(E)\, P_d(E')$$

Again ff the compound nucleus $[C]^\star$ is now formed in the same state of excitation by another process $X' + a'$, the cross section for disintegration into the above final state, ie $Z + d$, will be given by

$$\sigma(a', d) = \sigma_{a'}(E')\, P_d(E')$$

So dividing these equations we get

$$\frac{\sigma(a, b)}{\sigma(a, d)} = \frac{P_b(E')}{P_d(E')} = \frac{\sigma(a', b)}{\sigma(a', d)}$$

An experimental verification of the above relationship gives the direct test for the validity of Bohr's compound nucleus assumption.

4.6.3 Experimental Method

He studied the formation of compound nucleus Zn^{64} formed by α-bombardment of Ni^{60} and proton bombardment of Cu^{63}.

The exact α reaction channels were
$Ni^{60} + \alpha \rightarrow \left[Zn^{64}\right]^{\star} \rightarrow Zn^{63} + n$
$Ni^{60} + \alpha \rightarrow \left[Zn^{64}\right]^{\star} \rightarrow Zn^{62} + 2n$
$Ni^{60} + \alpha \rightarrow \left[Zn^{64}\right]^{\star} \rightarrow Cu^{62} + n + p$
The excitation curves were determined by stacked foil method. The α-excitation curves were obtained using the 40 MeV α-beam from the 60-inch cyclotron. In the case of the Nickel experiment, thin Ni foils of enriched Ni^{60} were prepared by electroplating the nickel on to copper; the copper was then dissolved by $AgNO_3$ solution. The abundance of Ni^{60} in the enriched sample was more than 85%.

The exact p reaction channels were
$Cu^{63} + p \rightarrow \left[Zn^{64}\right]^{\star} \rightarrow Zn^{63} + n$
$Cu^{63} + p \rightarrow \left[Zn^{64}\right]^{\star} \rightarrow Zn^{62} + 2n$
$Cu^{63} + p \rightarrow \left[Zn^{64}\right]^{\star} \rightarrow Cu^{62} + n + p$
Here also proton excitation curves were determined by same method by using the 32-MeV proton beam from the Berkeley linear accelerator. Ordinary Cu, consisting of Cu^{63} (69.1%) and Cu^{65} (30.9%) was used. Cu^{63} with proton forms Cu^{64} with a positron emission which is stopped by a 300 mg/cm^3 Al absorber which is a part of the stacked foil method. Whereas Cu^{65} with proton goes to Zn^{65} which has a halflife 250 days which is neglected in the context of the experimental time scale.

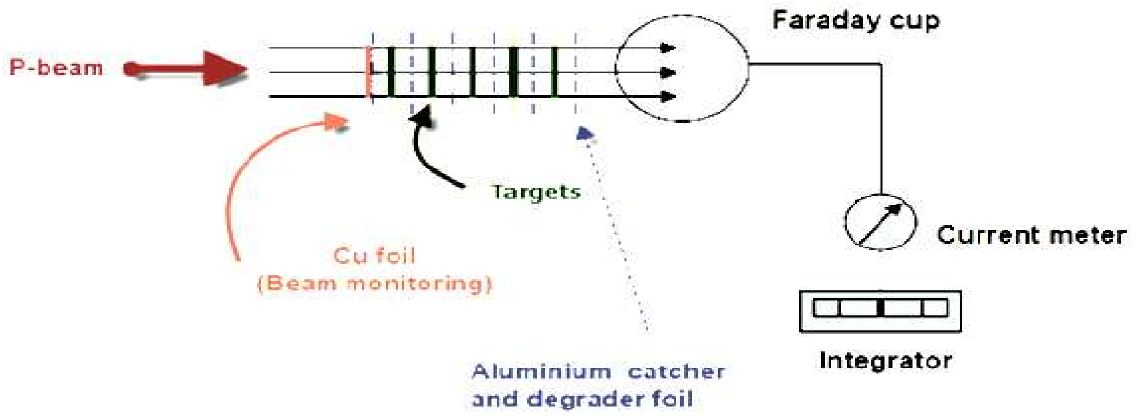
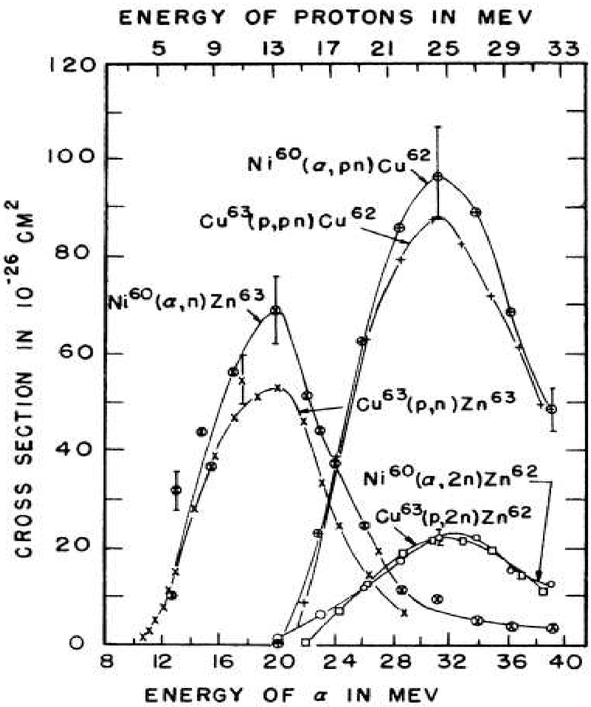


Figure 4.1: set for stacked foil method

4.6.4 Experimental Results

The experimental results are shown in Fig., where the observed cross sections for (α, n), (α, $2n$) and (α, pn) reactions on Ni^{60} and (p, n), (p, $2n$) and (p, pn) reactions on Cu^{63} are plotted as functions of the kinetic energy of the n-particles and protons respectively. The proton energy scale has been shifted by 7 MeV with respect to the alpha-energy scale in order to bring the peaks of the proton curves into approximate correspondence with those of the α-curves. This difference in energy to produce the same excitation is due to the difference in the masses of Cu^{63} + H^1 and Ni^{60} + He^4. It is clear from this figure that the ratios (α, n):(α, $2n$):(α, pn) for Ni^{60} agree, within the limits of experimental errors, with the ratios (p, n):(p, $2n$):(p, pn) for Cu^{63}. This agreement, according to the ratio relationships of cross-section, provides a direct test for the validity of the compound nuclear theory. *Plz note that the figure is directly taken form his paper which is referred as* ***Goshal, S. N. (1950). Phys. Rev. Vol. 80, 939***.



Chapter 5

Forces between Nucleons

5.1 Introduction

The interaction between two nucleons is basic for all of nuclear physics. The traditional goal of nuclear physics is to understand the properties of atomic nuclei in terms of the 'bare' interaction between pairs of nucleons. The oldest attempt to explain the nature of the nuclear force is due to Yukawa. According to his theory, π-mesons mediate the interaction between two nucleons. Although, in the light of quantumchromodynamics (QCD), meson theory is not perceived as fundamental anymore, the meson exchange concept continues to represent the best working model for a quantitative nucleon-nucleon (NN) potential. For the two-nucleon system, the experimental information consists of two particle scattering phase shifts in various partial waves, and the bound state properties of the deuteron. You know that the nuclear force resulted in nuclear interactions been nucleons have the following the main features:
- Attractive: to form nuclear bound states
- Short Range: of order 1 fm
- Spin-Dependent
- Noncentral: there is a tensor component
- Isospin Symmetric
- Hard Core: so that the nuclear matter does not collapse
- Spin-Orbit Force
- Parity Conservation

In phenomenology, one tries to come up with the forms of the forces which will satisfy the above properties. In particular, one parametrize the short distance potentials consistent with fundamental symmetries and fit the parameters to experimental data. For the system of two nucleons, we use $\vec{r} = \vec{r}_1 - \vec{r}_2$ to represent relative position and $\vec{p} = \dfrac{\vec{p}_1 - \vec{p}_2}{2}$ relative momentum and the total spin is $\vec{S} = \vec{s}_1 + \vec{s}_2$ with $\vec{s}_1$ and $\vec{s}_2$ being their indivisual spins. The relative orbital angular momentum is $\vec{L} = \vec{r} \times \vec{p}$. When the spins are coupled, the total spin can either be 0 or 1. For the case of $S = 0$, we have a single spin state which is called **singlet**. For the case of $S = 1$, we have three spin states which are called **triplet**. The total angular momentum is the sum of orbital angular momentum and total spin: $\vec{J} = \vec{L} + \vec{S}$, which involves the coupling of three angular momenta $\vec{s}_1$, $\vec{s}_2$ and $\vec{L}$. The orbital angular momentum quantum number is L. In the singlet spin case, we have $J = L$ because $S = 0$. For the triplet states, $J = L-1,\, L,\, L+1$. A state with (S, L, J) is usually labelled as ${}^{2S+1}L_J$, where $L = 0, 1, 2, 3..$ are usually called S, P, D, F, G... states. Since no nuclear interactions can couple states with different total angular momenta therefore J is a good quantum number instead of L or S. And the angular momemtum can be calculated from Clebsch-Gordon coefficicents. For the two nucleon system, the isospin T can either be 1 or 0. Since the total wave function has to be antisymmetric, therefore total symmetry factor is $(-1)^{L+S+T}$ which has to be -1. Therefore $L + T + S$ has to be odd. For deuteron, $S = 0$, $L = 0$ and therefore $T = 1$. The possible forms of the nuclear force could be a central one which just dependent on the relative distance $V_c(r)$. In this case, different L states have different energies. There could be also a pure spin-dependence force. The most general form is $V_s(r)\, \vec{\sigma_1} \cdot \vec{\sigma_2}$ with $\vec{\sigma_1} \cdot \vec{\sigma_2} = 2\,\vec{S}^2 - 3$. There can be also a pure iso-spin-dependence force. The most general form is $V_I(r)\, \vec{\tau_1} \cdot \vec{\tau_2}$ or else a spin-isospin dependent force given by $V_{sI}(r)\, \vec{\sigma_1} \cdot \vec{\sigma_2}\, \vec{\tau_1} \cdot \vec{\tau_2}$. Also may be there is inverse spin-orbit interaction, then the potential will take the form $V_{SL}(r)\, \vec{S} \cdot \vec{L}$. And finally there can be the worst case where the interaction can be tensorial too given by $V_T(r) \left[3\, \dfrac{(\vec{\sigma_1} \cdot \vec{r})\,(\vec{\sigma_1} \cdot \vec{r})}{r^2} - \vec{\sigma_1} \cdot \vec{\sigma_2} \right]$. Because of this peculiar type of nuclear force, major issues concerning the NN interaction, things which are investigated now-a-days are:
- charge-dependence
- the precise value of the πNN coupling constant
- improved phase shift analysis
- high-precision NN data with potentials
- QCD and the nuclear force
- NN scattering at intermediate and high energies.

5.2 Deuteron ${}_1D^2$: The simplest bound state system

The deuteron is made up of one neutron and one proton. It is relatively weakly bound (2.2245 MeV because of which the deuteron has no excited state; all observations on the deuteron are made on the ground state), but stable, and has a relative 0.015% natural isotopic abundance and is the only bound dinucleon. It can be easily understood that the diproton would be rendered unstable by Coulomb repulsion. Dineutrons do not exist in nature, not even as a short-lived unstable state. This is most likely due to the Pauli Exclusion Principle, the spin-spin interaction of the neutron and proton results in a Spin-1 system. Their magnetic moments have opposite signs to one another, hence the alignment of spins tends to antialign the magnetic dipoles, a more energetically stable configuration.
in a classical picture if you think of the neutron and proton as hard spheres that would be a tightly bound state in a classical picture and since the typical size of the nucleus is 1 Fermi the centre to centre distance would be 1 Fermi. This is a true picture, but it should do the job. Now, a loosely bound state is one where you do not expect a situation like this where they are as close as possible to each other. So, you would expect the root mean square size of the deuteron to be much more than a Fermi. And you would expect therefore, that the entire distance from there is more than 2 Fermi and then you would say it is loosely bound. The idea is to try to explain why the deuteron cannot be seen in an excited state. So, if it so loosely bound even a small perturbation, a small kick to the deuteron, a small supply of energy is enough to dissociate the neutron from the proton and separates the nucleus into the neutron and the proton separately, not keeping them as a bound state. Whereas, the amount of energy that is needed to kick it to a higher excited state is much more. And therefore, since even a small perturbation would separate the deuteron into a free neutron and proton that would explain: Why the deuteron is in a loosely bound state? Why the deuteron does not exist in the excited state?
Let us try to solve the deuteron problem quantum mechanically. Here the following assumptions are made to make the solution simple.
Assumptions:
- the interaction depends only on the distance between the two particles (and not for example the angle...)
- non-relativistic treatment to the problem
- spins have been neglected in the potential too.

The Hamiltonian is then given by the kinetic energy of the proton and the neutron and by their mutual interaction.

$$H = \frac{1}{2m_n}p_n^2 + \frac{1}{2m_p}p_p^2 + V(|r_p - r_n|)$$

where m_n and m_p are the mass of the neutron and proton respectively along with p standing for their corresponding momenta. We can solve the Schrodinger equation for the wavefunction $\Psi = \Psi(r_p, r_n, t)$. This is a wavefunction that treats the two particles as fundamentally independent (that is, described by independent variables). However, since the two particles are interacting, it might be better to consider them as one single system. Then we can use a different type of variables (position and momentum). Keeping in mind this we introduce a new set of variables namely R and r as the **average position** of the two particles (i.e. the position of the total system, to be accurately defined) and the **relative position** of one particle w. r. to the other as

$$R_{cm} = \frac{m_p r_p + m_n r_n}{m_p + m_n} \quad \text{center of mass,} \qquad r = r_p - r_n \quad \text{relative position}$$

Also, we can define the center of mass momentum and relative momentum (and velocity):

$$p_{cm} = \frac{m_p r_p - m_n r_n}{m_p + m_n}, \qquad p_r = p_p + p_n$$

Now we can also invert these equations and define indivisual masses and momenta by center of mass and relative variables. We also introduce the reduced mass of the system as $\mu = \dfrac{m_p\, m_n}{m_p + m_n}$ to make a dynamical system to behave as a static one. If you do the algebra to express these in terms of center of mass and relative variables, then the (classical) Hamiltonian, using these variables, reads, assuming $M = m_p + m_n$

$$H = \frac{1}{2M}p_{cm}^2 + \frac{1}{2\mu}p_r^2 + V(r)$$

Now, since the variables r and R_{cm} are independent (same as r_p and r_n) they commute. This is also true for p_{cm} and r (and p_r and R). Then, p_{cm} commutes with the whole Hamiltonian, $[p_{cm},\ H] = 0$. This implies that p_{cm} is a constant of the motion. This will be true for $E_{cm} = \dfrac{p_{cm}^2}{2M} = 0$ since we will be solving the problem in the center-of-mass frame which is not ever going to change. In general, it means that we can ignore the first term in the Hamiltonian and just solve

$$H = \frac{1}{2\mu}p_r^2 + V(r) = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r) \qquad \text{using} \quad p_{cm} = -i\,\hbar\,\frac{\partial}{\partial\, r}$$

$$H\,\Psi = -\frac{\hbar^2}{2\mu}\nabla^2\Psi + V(r)\Psi$$

Now this can be solved using separation of variables to the total Schrodinger equation which we have done in lots of time. Just remember that the Hamiltonian H (the deuteron Hamiltonian) is now the Hamiltonian of a single-particle system, describing the motion of a reduced mass particle in a central potential (a potential that only depends on the distance from the origin). This motion is the motion of a neutron and a proton relative to each other. All we don't know the exact shape of the potential. We just have assumed it as to be a central one. We need to write the Hamiltonian in spherical coordinates (for the reduced variables). The kinetic energy term is given by:

$$
\begin{aligned}
-\frac{\hbar^2}{2\mu}\nabla^2 &= -\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\,sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\,\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\,sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] \\
-\frac{\hbar^2}{2\mu}\nabla^2 &= -\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{\hbar^2}{2\mu}\left[\frac{1}{r^2\,sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\,\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\,sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] \\
&= -\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{L^2}{2\mu\,r^2}
\end{aligned}
$$

where we used the angular momentum operator (for the reduced particle) L^2 which have eigenvalues $l(l+1)\hbar^2$. Now the Schrodinger equation reads

$$
\left[-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{L^2}{2\mu\,r^2} + V(r)\right]\Psi_{n,l,m}(r,\theta,\phi) = E_n\,\Psi_{n,l,m}(r,\theta,\phi)
$$

Thus the wave function Ψ depends on three variables. The first piece depends only on r (the **radial part**), second piece on θ (the **polar part**) and the third piece on ϕ (the **azimuthal part**). Thus $\Psi_{n,l,m}(r,\theta,\phi)$ can be written as $\Psi_{n,l,m}(r,\theta,\phi) = \psi_{n,l}(r)\,\Theta_l(\theta)\,\varphi_m(\phi) = \psi_{n,l}(r)\,Y_l^m(\theta,\phi)$. In Quantum Mechanics, one often gets the θ and ϕ dependence packaged together in potentials as one function called a **spherical harmonics**. Then we can solve the Hamiltonian above with the separation of variables methods, or more simply look for a solution

$$
\begin{aligned}
\left[-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\,\psi_{n,l}(r)}{\partial r}\right)Y_l^m(\theta,\phi) + \psi_{n,l}(r)\frac{L^2Y_l^m(\theta,\phi)}{2\mu\,r^2}\right] &= [E_n - V(r)]\,\psi_{n,l}(r)\,Y_l^m(\theta,\phi) \\
\left[-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\,\psi_{n,l}(r)}{\partial r}\right)Y_l^m(\theta,\phi) + \psi_{n,l}(r)\frac{l(l+1)\hbar^2Y_l^m(\theta,\phi)}{2\mu\,r^2}\right] &= [E_n - V(r)]\,\psi_{n,l}(r)\,Y_l^m(\theta,\phi) \\
\left[-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\,\psi_{n,l}(r)}{\partial r}\right) + \psi_{n,l}(r)\frac{l(l+1)\hbar^2}{2\mu\,r^2}\right] &= [E_n - V(r)]\,\psi_{n,l}(r) \\
-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\,\psi_{n,l}(r)}{\partial r}\right) + \left[V(r) + \frac{l(l+1)\hbar^2}{2\mu\,r^2}\right]\psi_{n,l}(r) &= E_n\,\psi_{n,l}(r)
\end{aligned}
$$

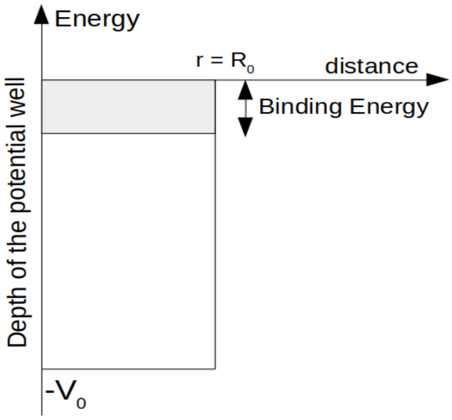
Now keeping in mind the well-behavedness of a wave function we write the $\psi_{n,l}(r)$ solution as $\frac{u_l(r)}{r}$ with boundary conditions that $u_{nl}(0) = 0 \rightarrow \Psi(0)$ is finite and $u_{nl}(\infty) = 0 \rightarrow$ leads to a bound state, as the variable r can take on only non-negative real values. Then the radial part of the Schrodinger equation becomes

$$
\begin{aligned}
-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}\left(\frac{u_l(r)}{r}\right)\right] + \left[V(r) + \frac{l(l+1)\hbar^2}{2\mu\,r^2}\right]\frac{u_l(r)}{r} &= E_n\,\frac{u_l(r)}{r} \\
-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left[-u + r\frac{\partial\,u}{\partial r}\right] + \left[V(r) + \frac{l(l+1)\hbar^2}{2\mu\,r^2}\right]\frac{u_l(r)}{r} &= E_n\,\frac{u_l(r)}{r} \\
-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\left[-\frac{\partial u}{\partial r} + \frac{\partial\,u}{\partial r} + r\frac{\partial^2\,u}{\partial r^2}\right] + \left[V(r) + \frac{l(l+1)\hbar^2}{2\mu\,r^2}\right]\frac{u_l(r)}{r} &= E_n\,\frac{u_l(r)}{r} \\
-\frac{\hbar^2}{2\mu}\frac{1}{r}\frac{\partial^2\,u}{\partial r^2} + \left[V(r) + \frac{l(l+1)\hbar^2}{2\mu\,r^2}\right]\frac{u_l(r)}{r} &= E_n\,\frac{u_l(r)}{r} \\
-\frac{\hbar^2}{2\mu}\frac{\partial^2\,u}{\partial r^2} + \left[V(r) + \frac{l(l+1)\hbar^2}{2\mu\,r^2}\right]u_l(r) &= E_n\,u_l(r) \\
\frac{\hbar^2}{2\mu}\frac{\partial^2\,u}{\partial r^2} + \left[E_n - V(r) - \frac{l(l+1)\hbar^2}{2\mu\,r^2}\right]u_l(r) &= 0 \\
\frac{\hbar^2}{2\mu}\frac{\partial^2\,u}{\partial r^2} + (E_n - V_{eff}(r))\,u_l(r) &= 0
\end{aligned}
$$

where $V_{eff}(r)$ is the **effective potential** that inputs the addition of a centrifugal potential (that causes an outward force). If l is large, the centrifugal potential is higher. The ground state is then found for $l = 0$ which is also energetically favored for a central potential. In that case there is no centrifugal potential and we only have a square well potential.

For $0 < r < R_0$

$$\frac{\partial^2 u}{\partial r^2} + \frac{2\mu}{\hbar^2}(E_n + V_0(r))\, u_l(r) = 0$$
$$\frac{\partial^2 u}{\partial r^2} + k_1^2\, u_l(r) = 0$$



which is simple harmonic in nature. We have also chosen a negative potential since we have assumed a square well potential as V_0. So the solution ie the eigenfunctions of this equation will be

$$\begin{aligned} u_{in}(r) &= A\sin(k_1 r) + B\cos(k_1 r) \\ &= A\sin(k_1 r) \end{aligned}$$

Using the boundary condition we can set $B = 0$ since at $r = \infty$ $u(r)$ should be zero.

For $r > R_0$

$$\frac{\partial^2 u}{\partial r^2} - \frac{2\mu}{\hbar^2} E_n\, u_l(r) = 0$$
$$\frac{\partial^2 u}{\partial r^2} - k_2^2\, u_l(r) = 0$$

which is exponential in nature. So the solution ie the eigenfunctions of this equation will be

$$\begin{aligned} u_{out}(r) &= C\, e^{-k_2 r} + D\, e^{k_2 r} \\ &= C\, e^{-k_2 r} \end{aligned}$$

Here also using the boundary condition we can set $D = 0$ since at $r = \infty$ $u(r)$ should be zero. Now these two solutions must match at $r = R_0$. So $\left[\frac{u'_{in}(r)}{u_{in}(r)}\right] = \left[\frac{u'_{out}(r)}{u_{out}(r)}\right]$ must be true.

$$\begin{aligned} \frac{A\sin(k_1 R_0)}{A\, k_1\cos(k_1 R_0)} &= \frac{-C\, k_2\, e^{-k_2 R_0}}{C\, e^{-k_2 R_0}} \\ k_1\cot(k_1 R_0) &= -k_2 \\ \cot(k_1 R_0) &= -\frac{k_2}{k_1} = -\frac{\sqrt{\frac{2\mu}{\hbar^2} E_n}}{\sqrt{\frac{2\mu}{\hbar^2}(E_n + V_0(r))}} = -\sqrt{\frac{E_n}{E_n + V_0(r)}} \end{aligned}$$

This equation can't be solved analytically. So numerical approach is to be made. However, we can get an idea about the minimum depth of the potential at the ground state of the deuterium just to have a bound state of n-p system. For that we will set $E_n = E_0 = 0$. So that we have

$$\cot(k_1 R_0) = 0 = \cot\left(\frac{\pi}{2}\right) \qquad \text{in general it's} \quad \cot\left[(2n+1)\,\frac{\pi}{2}\right]$$
$$\frac{2\mu}{\hbar^2}(E_n + V_0(r))\, R_0 = \frac{\pi}{2}$$
$$V_0 = \frac{\hbar^2}{2\mu}\,\frac{\pi^2}{4\, R_0^2} = 23.1 \text{ MeV}$$

which has been obtained after putting $R_0 = 2.1$ fm since the avarage size of proton and a neutron are almost in order of 1 fm and their masses are also almost equal so $\mu = \frac{M}{2}$. We thus find that indeed a bound state is possible, but the binding energy $E_0 = E_{kin} - V_0$ is quite small. Solving numerically the trascendental equation for E_0 we find that $E_0 = 2.2$ MeV. Notice that in our procedure we started from a model of the potential that includes the range R_0 and the strength V_0 in order to find the ground state energy (or binding energy). Experimentally instead we have to perform the inverse process. From scattering experiments it is possible to determine the binding energy (such that the neutron and proton get separated) and from that, based on our theoretical model, a value of V_0 can be inferred.

5.2.1 Excited state of ${}_1D^2$

Are bound excited states for the deuteron possible? We shall investigate the possibility of the exsistence of excited states of the deuteron. We will consider two possibilities.

Case I:
If we consider $n = 1$ in the equation $\cot\left[(2n+1)\frac{\pi}{2}\right]$ then we have $V_0 = \frac{9\,\hbar^2}{2\mu}\frac{\pi^2}{4\,R_0^2} = 9\,V_0 = 225$ MeV which violently disagrees in the n-p potential depth. For even higher values of n the disagreement is even more. So we conclude that no bound excited states of deuteron is possible for $n > 0$.

Case II:
If we choose $l = 1$ the quantity $\frac{l(l+1)\hbar^2}{2\,\mu\,R_0^2} \geq 18.75$ MeV. This means that the replusive centrifugal force tends to diminish the strength of the bindind energy of the deuteron. This will make the potential to become shallower (and narrower), and will be much lower than 23.1 MeV. So, even the second lowest value of l the system is no longer bound. **The deuteron has only one bound state**.

5.2.2 Spin and Parity of ${}_1D^2$

• **Spin**:
For calculation of spin we will use the shell model of nucleus. Deuteron has one proton and one neutron. So both of them can be placed in ${}^1s_{\frac{1}{2}}$ orbital. So contribution to the the spin coming from the proton is $\frac{1}{2}$ and that coming from neutron is also $\frac{1}{2}$. Hence this leads to the total spin of deuteron as $\frac{1}{2} + \frac{1}{2} = 1$

• **Parity**:
The parity of a state describes the behavior of its wave function under a reflection of the coordinate system through the origin. So theoritically separate the wave function into a product of three parts: **the intrinsic wave function of the proton**,**the intrinsic wave function of the neutron**, and **the orbital wave function for the relative motion between the proton and the neutron**. Since a proton and a neutron are just two different states of a nucleon, their intrinsic wave functions have the same parity. As a result, the product of their intrinsic wave functions has positive parity, regardless of the parity of the nucleon. This leaves the parity of the deuteron to be determined solely by the relative motion between the two nucleons. For states with a definite orbital angular momentum L, the angular dependence in the wave function is given by spherical harmonics $Y_{LM}(\theta\,\phi)$. Under an inversion of the coordinate system, spherical harmonics transform according to the relation

$$Y_{LM}(\theta\,,\phi) \longrightarrow Y_{LM}(\pi-\theta\,,\pi+\phi) = (-1)^L\,Y_{LM}(\theta\,,\phi)$$

The parity of $Y_{LM}(\theta\,\phi)$ is therefore $(-1)^L$. As the two nucleons is placed in ${}^1s_{\frac{1}{2}}$ orbital whose azimuthal quantum number l is zero. So parity P= $(-1)^{\sum l_p+\sum l_n}$ will lead to $(-1)^{0+0} = +1$ ie the parity is even.

5.2.3 Magnetic Dipole Moment of ${}_1D^2$

The magnetic dipole moment of a nucleus arises from a combination of two different sources. First, **each nucleon has an intrinsic magnetic dipole moment coming from the intrinsic spin and the orbital motion of quarks** and second since **proton carries a net positive charge, its orbital motion constitutes an electric current loop**. If, for simplicity, we assume that the proton charge is distributed evenly along its orbit, we can use classical electromagnetic theory to obtain its contribution to the magnetic dipole moment of a nucleus. Also it is more convenient to express the contributions to the nuclear magnetic dipole moment from individual nucleons in terms of gyromagnetic ratios. Thus the value of the nuclear magnetic moment is given by

$$\mu = g^{(l)}\,\vec{l} + g^{(s)}\,\vec{s} = \left[g_p^{(l)}\,\vec{l_p} + g_n^{(l)}\,\vec{l_n}\right] + \left[g_p^{(s)}\,\vec{s_p} + g_n^{(s)}\,\vec{s_n}\right]$$

Since only protons carry a net charge and, consequently, can contribute to the nuclear magnetic dipole moment coming from orbital motion part. Hence

$$\mu = g_p^{(l)}\,\vec{l_p} + \left[g_p^{(s)}\,\vec{s_p} + g_n^{(s)}\,\vec{s_n}\right] = \mu_N\,\vec{l_p} + \frac{1}{2}(5.58569\,\mu_N - 3.826085\,\mu_N) \qquad \text{since } s = \frac{1}{2} \text{ and } \quad \mu_N = \frac{e\,\hbar}{2m}$$

Since the masses of a proton and a neutron are roughly equal to each other, we may assume that each one of the two nucleons carries one-half the orbital angular momentum associated with their relative motion, i.e., $l_p = \frac{1}{2}L$ where L is the deuteron orbital angular momentum operator. Thus the final result is then

$$\mu = \mu_N\,\frac{1}{2}\,L + \frac{1}{2}(5.58569\,\mu_N - 3.826085\,\mu_N)$$

But for triplet state ${}^3S_1$, $L = 0$ and the expectation value of the magnetic dipole operator reduces to a sum of the intrinsic dipole moments of a proton and a neutron, and hence we have $\mu = 0.879805\,\mu_N$. But the experimentally observed value is found out to be $0.857438\,\mu_N$. This difference needs more careful consideration. Orbital angular momentum $l = 0$ and $l = 2$ give the correct parity determined from experimental observations. Thus The observed even parity allows us to consider both $l = 0$ and $l = 2$ as possibilities which means not only ${}^3S_1$ for $l = 0$ is the lone contributor but contribution is coming also from ${}^3D_1$ for $l = 2$. We can make a simple estimate of the amount of ${}^3D_1$-component in the deuteron ground state using the measured value of μ and the calculated values of $\mu({}^3S_1)$ and $\mu({}^3D_1)$ obtained above. For a linear combination of ${}^3S_1$ and ${}^3D_1$-components, the deuteron wave function may be written as

$$|\psi\rangle = a_S \left|{}^3S_1\right\rangle + a_D \left|{}^3D_1\right\rangle$$

with the normalisation condition that $a_S^2 + a_D^1 = 1$. Since there is no off-diagonal matrix element of μ between ${}^3S_1$- and ${}^3D_1$-states, the value of a_S^2 and a_D^2 is found to be 0.96 and 0.04. This means that the deuteron is 96% is in $l = 0$ (s orbit) and only 4% is in $l = 2$ (d orbit) which has been the admixture of the ${}^3D_1$-component in the deuteron ground state.

5.2.4 Electric Quadrupole Moment of ${}_1D^2$

The deuteron was found to possess an electric quadrupole moment in 1939. This discovery had far reaching consequences: it meant that nuclear forces were not central and were more complex that previously thought. In contrast to the case of the magnetic moment which is determined through its coupling to an external applied magnetic field, the quadrupole moment does not couple to an external electric field. One measures the interaction of the deuteron quadrupole moment with the electric field gradient created along the molecular axis by the neighbouring atom.
For a spherical nucleus, the expectation values of the squares of the distance from the center to the surface along x-, y-, and z-directions are equal to each other

$$\langle\, x^2 \,\rangle = \langle\, y^2 \,\rangle = \langle\, z^2 \,\rangle$$

As a result the expectation value $\langle r^2\rangle = \langle x^2 + y^2 + z^2\rangle = 3\,\langle\, z^2 \,\rangle$. The electric quadrupole operator, which measures the lowest order departure from a spherical charge distribution in a nucleus, is defined in terms of the difference between $3\,Z^2$ and r^2 ie $Q_0 = e\,\left(3\,Z^2 - r^2\right)$. In terms of spherical harmonics this is given as

$$Q_0 = e\,\left(3\,Z^2 - r^2\right) = e\,r^2\,(3\,cos^2\theta - 1) = \sqrt{\frac{16\pi}{5}}\,e\,r^2\,Y_{20}(\theta\,\phi)$$

The electric quadrupole moment of a nuclear state is defined as the expectation value of Q_0 in the substate of maximum M. But Q_0 operates only in the coordinate space, it is independent of the total intrinsic spin S. This means that the orbital angular momentum L of the state must also be greater or equal to 1 which in turn indicates ${}^3S_1$ state doesn't contribute to the electric quadrupole moment and only ${}^3D_1$ state will. To examine further we need to evaluate the radial integrals, so we would need to solve the radial Schrodinger equation and obtain the radial wave functions. Clearly, for a given potential model this is (in principle) possible ie we cannot evaluate its value without making some assumptions on the radial wave function. If, as an assumption, we take the value of $\langle\, r^2 \,\rangle$ as the square of the deuteron radius, we obtain $Q_0 = -0.77e$ fm^2. Since the expectation value have to be always positive and the sign disagrees with the measured value, it is unlikely that the deuteron wave function is made up entirely of the ${}^3D_1$-state. For a more realistic model, we need to take a linear combination of ${}^3S_1$- and ${}^3D_1$-components. This will give us an approximate expression that we can set equal to the experimental value $Q_{exp} = 0.286e$ fm^2. This value seems quite reasonable given that the mean squared charge radius of deuteron is 4 fm^2.

Chapter 6

Nuclear Instrumentation

6.1 Particle Accelerator

You have probably read about or heard of particle accelerators in numerous scientific discussions, especially those pertaining to particle physics. For the record, they deserve more attention than they get! For example, the Large Hadron Collider (LHC) a particle accelerator is the single largest machine ever built by mankind. That staggering fact might make you wonder what is it actually? And perhaps more importantly, why should I care what it does? Let me put it this way. Did you know that you have a type of particle accelerator in your house right now? The cathode ray tube (CRT) of any TV or computer monitor is really a particle accelerator. *But now a days LCD monitors are in the market. See old days are gone.* The CRT takes particles (electrons) from the cathode, speeds them up and changes their direction using electromagnets in a vacuum and then smashes them into phosphor molecules on the screen. The collision results in a lighted spot, or pixel, on your TV or computer monitor. A particle accelerator works the same way, except that they are much bigger, the particles move much faster (near the speed of light) and the collision results in more subatomic particles and various types of nuclear radiation. Particles are accelerated by electromagnetic waves inside the device, in much the same way as a surfer gets pushed along by the wave. The more energetic we can make the particles, the better we can see the structure of matter. It's like breaking the rack in a billiards game. *Think about your 8 ball pool installed in your android phone.* When the cue (the white) ball (energized particle) speeds up, it receives more energy and so can better scatter the rack of balls (release more particles).
The formal Definition:
A particle accelerator is a machine that accelerates elementary particles, such as electrons or protons, to very high energies. On a basic level, particle accelerators produce beams of charged particles that can be used for a variety of research purposes.

6.1.1 Types of accelerators

Particle accelerators come in two basic types:
- **Circular** - Particles travel around in a circle until they collide with the target.
- **Linear** - Particles travel down a long, straight track and collide with the target.

6.2 The Cyclotron

Let's now discuss one of the most fundamental and earliest of accelerators, the cyclotron which is still used as the first stage of some large multi-stage particle accelerators. It is a device used to accelerate charged particles like protons, deutrons, α-particles, etc, to very high energies. It was invented by **Ernest O. Lawrence** in 1929-1930 at the University of California, Berkeley and patented in 1932. Lawrence received the 1939 Nobel prize in physics for this work.

6.2.1 Principle

A charged particle can be accelerated to very high energies by making it pass through an electric field a number of times. *So if question comes whether a neutron can be accelerated or not and the answer is "no" since neutron is chargless.* This can be done with the help of a perpendicular magnetic field which throws the charged particle into a circular motion, the frequency of which does not depend on the speed of the particle and the radius of the circular orbit. Thus electric field is used to accelerate in a translational manner and magnetic field is it used to make the charge particle to go it in a circular path. Below are the two figures of it. Left is a schematic diagram and right is 60-inch cyclotron at Berkeley's Rad Lab. Ernest Lawrence is second from the left if you can figure him out.

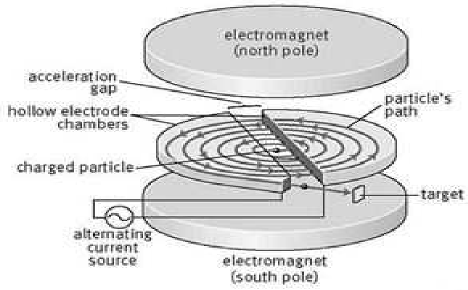


Fig: schematic diagram of a cyclotron

6.2.2 Construction

• It consists of two small, hollow, metallic half-cylinders D_1 and D_2 called dees as they are in the shape of D which are put back to back. (*like your bicycle chain ring of the padel*)
• They are mounted inside a vacuum chamber (*the whole device is in high vacuum (pressure* $\sim 10^{-6}$ *mm of Hg) so that the air molecules may not collide with the charged particles*) between the poles of a powerful electromagnet.
• The dees are connected to the source of high frequency alternating voltage of few hundred kVs (*depending upon your need*). Thus theses dees are acting as electrodes.
• The beam of charged particles to be accelerated is injected into the dees near their centre, in a plane perpendicular to the magnetic field.
• The charged particles are pulled out of the dees by a deflecting plate through a window to collide with the target.

6.2.3 Theory

Suppose a positive ion,say a proton,enters the gap between the two dees and finds dee D_1 to be negative.It gets accelerated towards dee D_1.As it enters the dee D_1,it does not experience any electric field due to shielding effect of the metallic dee. The perpendicular magnetic field throws it into a circular path. At the instant the proton comes out of dee D_1,,it finds dee D_1 positive and dee D_2. It moves faster through D_2 describing a larger semicircle than before. Thus if the frequency of the applied voltage is kept exactly the same as the frequency of revolution of the proton,then every time the proton reaches the gap between the two dees, the electric field is reversed and proton receives a push and finally it acquires very high energy. This is called the cyclotrons resonance condition. The proton follows a spiral path. The accelerated proton is ejected through a window by a deflecting voltage and hits the target.
Let a particle of charge q and mass m enter a region of magnetic field $\vec{B}$ with a velocity $\vec{v}$ normal to the field $\vec{B}$. The particle follows a circular path of radius r, the necessary centripetal force begin provided by the magnetic field. Therefore,

$$\begin{aligned}
\text{Centripetal force on charge q} &= \text{Magnetic force on charge q} \\
\frac{m\,v^2}{r} &= B\,q\,v \\
\frac{v}{r} = \omega &= \frac{B\,q}{m} \\
\nu &= \frac{B\,q}{2\,\pi\,m}
\end{aligned}$$

Clearly,this frequency is independent of both the velocity of the particle and the radius of the orbit and is called **cyclotron frequency** or **magnetic resonance frequency**. This is the key fact which is made use of in the operation of a cyclotron. Thus as the beam spirals out, the frequency doesn't decrease and it must continue to accelerate as it is travelling more and more distance at the same time. As the beam spirals out and thus acquiring higher and higher velocities just before coming out the dees it attains the maximum velocity and thus with maximum kinetic energy. Hence

$$\begin{aligned}
\frac{m\,v_{max}^2}{r_{max}} &= B\,q\,v_{max} \\
v_{max} &= \frac{B\,q\,r_{max}}{m} \\
\frac{1}{2}\,m\,v_{max}^2 = \mathrm{KE}_{max} &= \frac{1}{2}\,\frac{B^2\,q^2\,r_{max}^2}{m}
\end{aligned}$$

6.2.4 Utility

- The high energy particles produced in a cylinder are used to bombard nuclei and study the resulting nuclear reactions and hence investigate nuclear structure.
- It is used to implant ions into solids and modify their properties or even synthesis new materials.
- It is used to produce radioactive isotopes which are used in hospitals for diagnosis and treatment.

6.2.5 Drawbacks

- The significant drawback that the cyclotron suffers is from the effect of relativistic mass. According to the Einsteins special theory of relativity,the mass of a particle increases with the increase in its velocity as $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ where m_0 is the rest mass of the particle. At high velocities,the cyclotron frequency will decrease due to increase in mass. This will throw the particles out of resonance with the oscillating field. That is, as the ions reach the gap between the dees, the polarity of the dees is not reversed at that instant. Consequently the ions are not accelerated further. That's a serious thing people are concerned with.
- Electrons cannot be accelerated in a cyclotron. A large increase in their energy increases their velocity to a very large extent. This throws the electrons out of step with the oscillating field.

6.3 The Linear Accelerator : LINAC

In 1924 Gustaf Ising, a Swedish physicist, proposed accelerating particles using alternating electric fields, with drift tubes positioned at appropriate intervals to shield the particles during the half-cycle when the field is in the wrong direction for acceleration. Four years later, the Norwegian engineer Rolf Wideroe built the first machine of this kind, successfully accelerating potassium ions to an energy of 50,000 electron volts (50 keV).

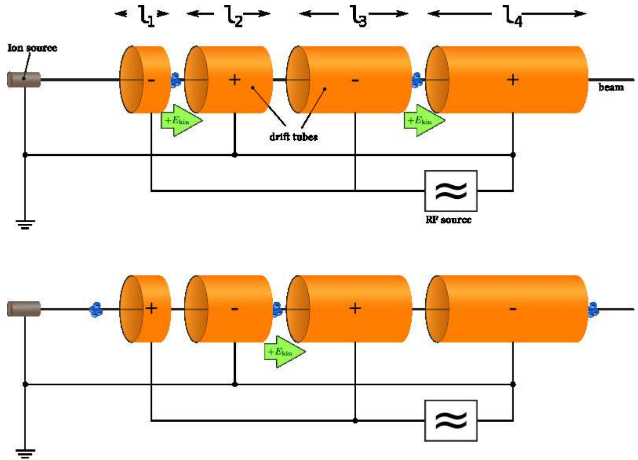
6.3.1 Principle

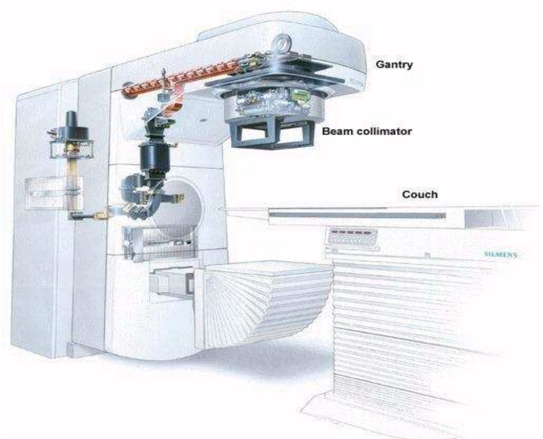
This type of particle accelerator that imparts a series of relatively small increases in energy to subatomic particles as they pass through a sequence of alternating electric fields set up in some hollow tubes of variable lenghts arranged in a linear manner. The small accelerations acquired by particles gets added together each time they come out of the tube to give rise to a greater energy than could be achieved by the voltage used in one tube alone.

6.3.2 Construction

The exact design of a LINAC depends on the type of the particle that is being accerlerated: electrons, protons or ions. But the basic necessities are the following

- A **particle gun** is located at the left in the drawing. If the particle is e^- then a cold cathode or a hot cathode or a photodiode is placed. For protons an ion source is needed. If heavier particles are to be accelerated (e.g. Uranium ions) a specialised ion source is placed.
- A **high voltage source** is required to emit particles out of the particle gun ie for initial injection of particles.
- **Hollow evacuated cylindrical tubes**, also called as **drift tubes** are used to accelerate the particles which are coming out of the particle gun and also pack them into bunches. Thus these tubes are also sometimes called as **buncher**. However the lengths will vary with the application.
- A **radio frequency energy source** is needed to energize the cylindrical tubes which will then behave as electrodes so that particles can accelerate.





6.3.3 Theory

As the theme is accelerating charge particle therefore particles are kicked by a radio-frequency alternating electric fields that are applied over short distances which is applied between the drift tubes (across the gap between them). As the particles leave a tube there must be an accelerating field across the gap so the tube it is in must be of the same potential as the particle's charge (repelling it away from where it has come from) and the one it is about to enter be of the opposite potential (attracting it towards the next tube). However inside the tube the particles do not feel any force because of effect called as **Faraday's cage** which states that electric charge on a conductor sits on the outer surface of it or electric field inside a conductor is always zero. (*That's why you should not come out of car while driving during thunderstorm*). The passage of the particle between drift tubes is synchronized with the phase of the accelerating field - the particle is only subjected to the field when it is in the part of the cycle that accelerates it. In other words the time required for particles to pass through any tube is exactly made equal with frequency of the polarity reversal of the field.
Let a particle of charge q and mass m enter a region of electric field where the particle is accelerated by a potential V. Now the kinetic energy acquired by the particle is

$$\frac{1}{2}\,m\,v^2 = V\,q$$

But this amount of KE will be attained only in gaps between the tubes. Inside the tube the KE will remain fixed as the tube will serve as Faraday's cage. If there are n numbers of such tubes between each gap that much amount of KE will be added everytime. Thus after passing through n tubes the KE will increase to a larger extent. Let v_n is the vel. after the nth tube. Then

$$\frac{1}{2}\,m\,v_n^2 = n\,V\,q$$

$$v_n = \sqrt{\frac{2\,n\,V\,q}{m}}$$

Thus the velocities after coming out of each tube are in the ratio of $v_1 : v_2 : v_3 : v_4 : \ldots\ldots = 1 : \sqrt{2} : \sqrt{3} : \sqrt{4} : ....$
If L_n is the length of such nth tube and the particles need a time t to cross that then

$$L_n = v_n\,t = v_n\,\frac{1}{2\,\nu} = \sqrt{\frac{2\,n\,V\,q}{m}}\,\frac{1}{2\,\nu} \qquad \text{where } \nu \text{ is the frequency of the oscillator}$$

Thus you can also see that the length of the tubes are also in the ratio of $L_1 : L_2 : L_3 : L_4 : ..... = 1 : \sqrt{2} : \sqrt{3} : \sqrt{4} : ....$

6.3.4 Utility

- In the linac, the particles are accelerated multiple times by the applied voltage and hence used to study matter-antimatter annihilation and in production of radio-isotope used in medical purposes.
- Linac-based radiation therapy is used in cancer therapy and in treatment of benign and malignant disease.

6.3.5 Drawbacks

- The device length limits the locations where one may be placed.
- A great number of devices and their associated power supplies are required, increasing the construction and maintenance expense of this portion.

6.4 LINAC vs Cyclotron

Let me put it straight in a tabular form.

LINAC	**Cyclotron**
Large space requirement but light	Compact but heavy
Very Expensive	Relatively Cheaper
Upgradable in energy	Difficult to upgrade in energy
Straightforward beam extraction	Difficult extraction of beam

Chapter 7

Nuclear Detectors

7.1 Introduction

Nuclear radiation detectors serve to determine the composition and measure the intensity of radiation, to measure the energy spectra of particles, to study the processes of interaction between fast particles and atomic nuclei, and to study the decay processes of unstable particles. Interactions of α, β and γ radiations with matter may produce positively charged ions and electrons. The detectors are devices that measure this ionisation and produce and produce an observable output. Early detectors used photographic plates to detect "tracks" left by nuclear interactions. Advances in electronics, particularly the invention of the transistor, allowed the development of electronic detectors. Advances in materials, particularly ultra-pure materials, and methods of fabrication have been critical to the creation of new and better detectors. All of these have increased the accuracy of measurements and also the efficiency of detectors.

7.1.1 Classification of detectors

We may conveniently classify the detectors into two classes
- Electrical detectors
- Optical detectors

Table 7.1: Classification of detectors with examples

Electrical	**Optical**
Ionization Chamber	Cloud Chamber
Proportional Counter	Bubble Chamber
Geiger-Muller Counter	Spark Chamber
Scintillation Counter	Photographic Emulsion
Cerenkov Counter	
Semi-conductor detector	

7.1.2 Efficiency of detectors

An important characteristic of nuclear radiation detectors that register individual particles is their efficiency - the probability of the registration of a particle upon entry into the effective volume of the detector. Efficiency is a function of the design of the detector and the properties of the working medium. However according to Hofstadter a perfect detector might have the following characteristics

- 100% detection efficiency. (*No events should be missied out*)
- High-speed counting (*More quickly it detects, better for us*)
- Good energy resolution (*Two events even with small energy differences should be measured. That means they have to counted two instead of one.*)
- Linearity of response (*More radiation produced more should be the detection*)
- Application to virtually to all types of particles and radiations (*One single detector should capable of detecting all types of particles. Though it's not possible.*)
- Virtually no limit to the highest energy detectable (*This is also highly anticipated for. It will be very nice to design a detector which can detect particles even in GeV range or higher*)
- Reasonably large solid angles of acceptance (*The detector should be rotated in all possible direction so that no event gets missed.*)

- Discrimination between types of particles. (*All particles should be classified once it detected.*)
- Picturization of the event. (*A photo will be a nice thing to upload in facebook.*)

7.2 Cloud Chamber

7.2.1 Introduction

A cloud chamber makes the invisible visible, allowing us to see delicate, wispy proof that there are tiny particles whose story starts in outer space shooting through all of us, every minute of every day. It's a unique device for detection and measurement of elementary particles and other ionizing radiation. Also known as a Wilson Cloud Chamber after the name of inventor C.T.R. Wilson in 1911. In particular, the discoveries of the positron in 1932 and the muon in 1936, both by Carl Anderson (awarded a Nobel Prize in Physics in 1936), used cloud chambers. Discovery of the kaon by George Rochester and Clifford Charles Butler in 1947, also was made using a cloud chamber as the detector.

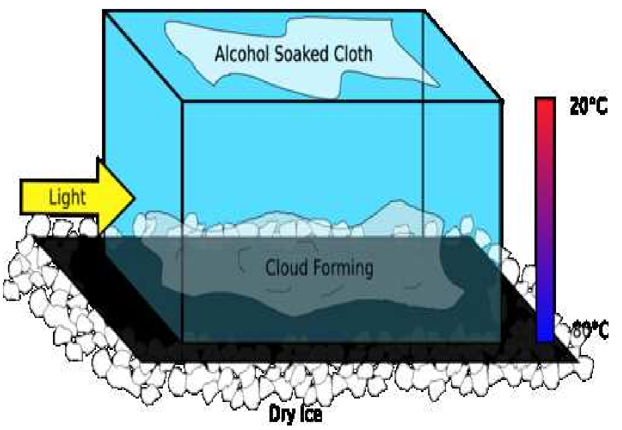
7.2.2 Construction

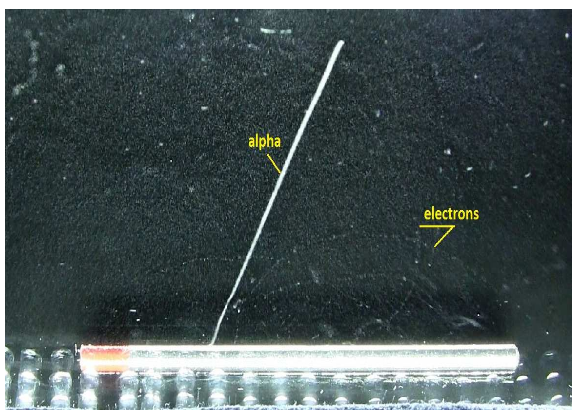
The construction of the cloud chamber is very simple and naive one. You can make it in your home also. (*But I doubt if you could detect a particle in that*) Here is what you need to construct it
- A closed chamber.(*Say a fish tank ie an aquarium of any shape*)
- Some alchohol (*Go to the chemistry department. I never said to go to a wine shop.*)
- Some dry ice. (*Go to the daily bazaar and ask in the fish seller.*)
- A perforated substance. (*Again chemistry department. One iron net like that net which you use while heating something in Bunsen burner.*)
- One piece of cloth to put alchohol.
- A hot source. (*A hot water bag will also work*)
- One black piece of cloth to cover up the entire set up.
- One light source (*A simple torch. Don't use mobile phone light. You will need some brightness.*)

All you have to maintain inside the chamber a temperature gradient and a supersaturated environment. Temperature gradient from top to bottom. So bottom of the chamber is to be kept cool and top of the chamber is to be kept hot. This is why the dry ice is kept at the bottom of the chamber the hot source is placed is placed at the top. But just below the hot source the perforated substance is kept upon which there lies the piece of cloth and over that piece of cloth plenty of alchohol is poured. This hot source will evaporate the alchohol inside the chamber since alchohol is a volatile substance. As the vapour falls, it cools rapidly due to the dry ice and the air becomes supersaturated and after a while the entire chamber will become supersaturated with alchohol vapour.

7.2.3 Working Principle

Now let us consider a charged particle (such as α radiation from a chunk of radioactive ore) zips through the chamber at high speed. It bumps into alcohol molecules and ionizes them - it creates a trail of ionized molecules marking its path. Now, the vapours are such that they really want to produce mist; The trail of ionized molecules is enough to do that - the ions attract a bunch of molecules, the resulting clumps attract even more, and before you know it a droplet of alchohol is formed, then another, and another. Well, a trail of mist follows the particle. However, these droplets are visible as a "cloud" track that persist for several seconds while the droplets fall through the vapor which can be better seen by a tangential application of a light source. Then how identify which particle's tract they are? Well, the tracks have characteristic shapes. For example, an α particle track is thick and straight, while an electron track is wispy and shows more evidence of deflections by collisions.





7.3 Ionisation Chamber

7.3.1 Introduction

The ionization chamber is the simplest of all gas-filled radiation detectors, and is widely used for the detection and measurement of certain types of ionizing radiation; X-rays, γ rays, and β particles. The term "ionization chamber" is used exclusively to describe those detectors which collect all the charges created (ie the current) by direct ionization within the gas through the application of an electric field. In an ideal case, the amount of electric current generated in an ionization chamber is directly proportional to the intensity of the radiation field. Thus the ionisation chambers have a good uniform response to radiation over a wide range of energies and hence finds application in the nuclear power industry, research labs, radiography, radiobiology, and environmental monitoring.

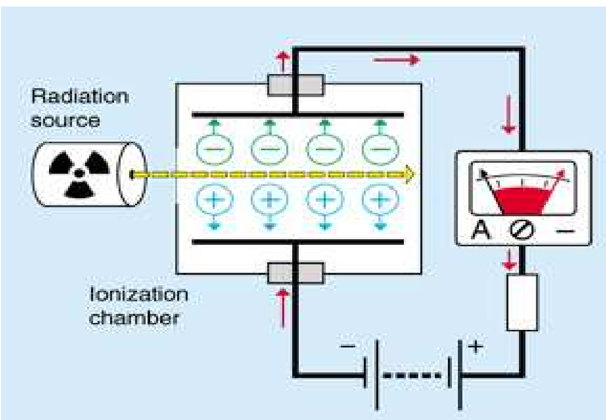
7.3.2 Construction

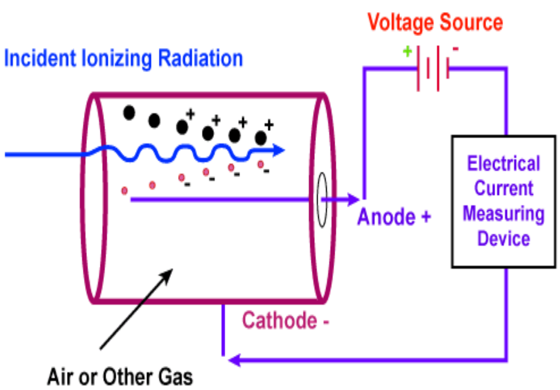
It's construction is also very simple but vastly modified as compared to cloud chamber. Following are the basic needs to construct an ionisation chamber.

- Two collecting electrodes: the anode and cathode (the anode is positively charged with respect to the cathode). In most cases, the outer chamber wall serves as the cathode. The electrodes may be in the form of parallel plates (Parallel Plate Ionization Chambers: PPIC), or a cylinder arrangement with a coaxially located internal anode wire. (*Just think of this you have a bottle of cold drink behaving as cathode and you insert the straw to suck the drinks behaving as the anode*)
- A voltage source (*This will create an electric field between the electrodes*)
- An electrometer circuit (*This is capable of measuring the very small output current which is in the region of femtoamperes to picoamperes*)

7.3.3 Working Principle

First the potential difference between the anode and cathode is often in the 100 to 500 volt range. The most appropriate voltage depends on a number of things such as the chamber size (the larger the chamber, the higher the required voltage). When an ionising radiation or a charged particle enters the chamber, it converts some of the gas molecules to positive ions and electrons; under the influence of the electric field, these particles migrate to the wall and the wire, respectively, and cause an observable current to flow through the circuit. This accumulated charge is proportional to the number of ion pairs created, and hence implies the strength the radiation dose which is a measure of the total ionizing dose entering the chamber. However there is one problem with this set up. As the produced electrons move toward the anode, on its journey it may recombine with other ions to produce a neutral element. Thus there is posibility that the ion current will diminish due to recombination. Thus it can be seen that in the "ion chamber" operating region the collection of ion pairs is either effectively constant or less than the expected value over a range of applied voltage, as due to its relatively low electric field strength.





7.4 Proportional Counter

7.4.1 Introduction

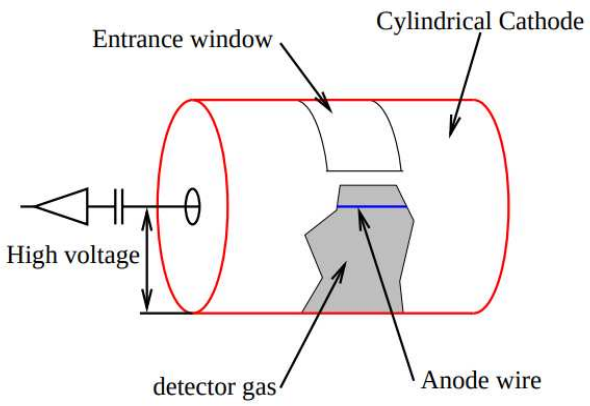
The proportional counter is a type of gaseous ionization detector device used to measure particles of ionizing radiation. The key feature is its ability to measure the energy of incident radiation, by producing a detector output pulse that is proportional to the radiation energy absorbed by the detector due to an ionizing event, hence the detector's name. It is widely used where discrimination between radiation types is required, such as between alpha and beta particles.

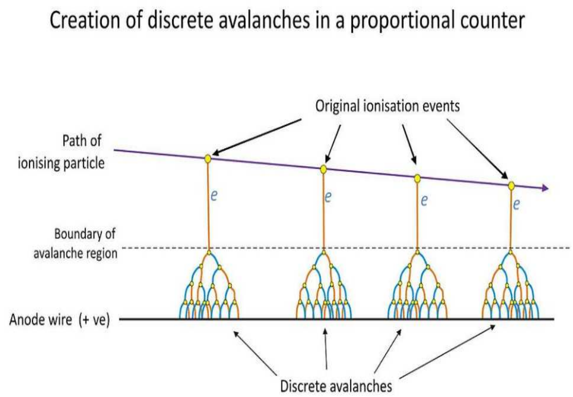
7.4.2 Construction

A proportional counter is much advanced version of an ionisation chamber, and operates in a voltage region more than ionisation chamber.
• The anodes are usually thin metal wires, and their electric field causes the electrons to drift towards the anodes where the field strength is highest. Anodes in the detector volume are held at a positive potential with respect to the rest of the detector.
• The cathode is cylinder arranged in a co-axial manner. The metal wire is at the center surrounding that the cathode cylinder.
• A voltage source (*This will create an electric field between the electrodes*)
• An electrometer circuit (*This is capable of measuring the very small output current which is in the region of femtoamperes to picoamperes*)

7.4.3 Working Principle

In a proportional counter the fill gas of the chamber is an inert gas which is ionised by incident radiation. An ionizing particle entering the gas collides with a molecule of the inert gas and ionises it to produce an electron and a positively charged atom, commonly known as an "ion pair". As the charged particle travels through the chamber it leaves a trail of ion pairs along its trajectory, the number of which is proportional to the energy of the particle if it is fully stopped within the gas. The chamber geometry and the applied voltage is such that in most of the chamber the electric field strength is strong enough to prevent re-combination of the ion pairs and causes positive ions to drift towards the cathode and electrons towards the anode. This is the "ion drift" region. In the immediate vicinity of the anode wire, the field strength becomes large enough to produce Townsend avalanches. This avalanche region occurs only fractions of a millimeter from the anode wire, which itself is of a very small diameter. The purpose of this is to use the multiplication effect of the avalanche produced by each ion pair. This is the "avalanche" region. Therefore it can be said that the proportional counter has the key design feature of two distinct ionisation regions:
• *Ion drift region:* in the outer volume of the chamber - creation of number ion pairs proportional to incident radiation energy.
• *Avalanche region:* in the immediate vicinity of the anode - Charge amplification of ion pair currents, while maintaining localised avalanches.
In summary, the proportional counter is an ingenious combination of two ionisation mechanisms in the one chamber which greatly improves the signal-to-noise ratio of the detector and hence finds wide practical use.





7.5 Geiger-Muller Counter, (GM counter)

7.5.1 Introduction

It is an instrument used for detecting and measuring ionizing radiation, α, β and γ radiation. The principle of working remains the same as that of proporsonal counter, charged particles ionize the gas through which they pass ,the electrons so produced during ionization get accelerated under high potential and further produce ionization. The main advantages are that they are relatively inexpensive, durable and easily portable. But they have very low efficiency in determining the the exact energy of the detected radiation.

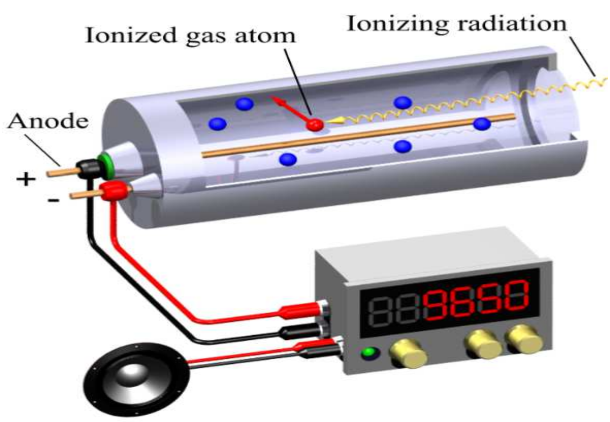
7.5.2 Construction

The construction of the GM counter is exactly similar to that of the proporsonal counter. A Geiger tube which is nothing but a charged capacitor with a region between them occupied by a gas. The apparatus consists of two parts, the tube and the (counter + power supply). The Geiger-Mueller tube is usually cylindrical, with a wire down the center. The (counter + power supply) have voltage controls and timer options. A high voltage is established across the cylinder and the wire.

- The anodes are usually thin metal wires, which are held at a positive potential with respect to the rest of the detector.
- The cathode is cylinder arranged in a co-axial manner. The metal wire is at the center surrounding that the cathode cylinder.
- A voltage source (*This will create an electric field between the electrodes*)
- An electrometer circuit (*This is capable of measuring the very small output current which is in the region of femtoamperes to picoamperes*)

7.5.3 Working Principle

When ionizing radiation such as an α, β or γ particle enters the tube, it can ionize some of the gas molecules in the tube. From these ionized atoms, an electron is knocked out of the atom, and the remaining atom is positively charged. The high voltage in the tube produces an electric field inside the tube. The electrons that were knocked out of the atom are attracted to the positive electrode, and the positively charged ions are attracted to the negative electrode. This produces a pulse of current in the wires connecting the electrodes, and this pulse is counted. After the pulse is counted, the charged ions become neutralized, and the Geiger counter is ready to record another pulse. In order for the Geiger counter tube to restore itself quickly to its original state after radiation has entered, a gas is added to the tube. This gas is called as a quench gas to ensure each pulse discharge terminates; a common mixture is 90% argon, 10% methane. For low voltages, no counts are recorded. This is because the electric field is too weak for even one pulse to be recorded. As the voltage is increased, eventually one obtains a counting rate. The voltage at which the G-M tube just begins to count is called the starting potential. The counting rate quickly rises as the voltage is increased. The rise is so fast, that the graph looks like a step potential. After the quick rise, the counting rate levels off. This range of voltages is termed the plateau region. Eventually, the voltage becomes too high and we have continuous discharge. The threshold voltage is the voltage where the plateau region begins. Proper operation is when the voltage is in the plateau region of the curve.



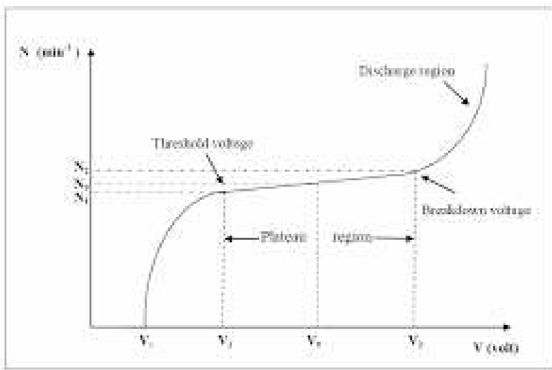


Fig.1

- **Dead Time**: After a count has been recorded, it takes the G-M tube a certain amount of time to reset itself to be ready to record the next count. The resolving time or "dead time", T, of a detector is the time it takes for the detector to "reset" itself. Since the detector is "not operating" while it is being reset, the measured activity is not the true activity of the sample. If the counting rate is high, then the effect of dead time is very important.

Dr. Upakul Mahanta, Department of Physics, Bhattadev University, Bajali

Chapter 8

Elementary Particles

8.1 Introduction

Elementary particles, the most basic physical constituents of the universe. The physics of it deals with the fundamental constituents of matter and their interactions. The particle physicists differ from other physicists in the scale of the systems that they study. They are interested in physical processes that occur at scales even smaller than atomic nuclei. At the same time, they engage the most profound mysteries in nature: How did the universe begin and have evolved so far? What explains the pattern of masses in the universe? Why is there more matter than antimatter in the universe etc. In the past several decades an enormous amount of experimental information has been accumulated, and many patterns and systematic features have been observed. Highly successful mathematical theories of the electromagnetic, weak, and strong interactions have been devised and tested and the results are speculative but encouraging towards the unification of these interactions into a simple underlying framework,**theory of everything**.

Some basic definition

• **Elementary particle:** a fundamental constituent of matter.
• **Antiparticle:** a counterpart of a given particle with the same mass but opposite charge and magnetic moment.
• **Baryon:** a family, or spectrum of heavy particles, the ground state of which is the proton.
• **Lepton:** a family of "light particles" which as a class, together with the photon, does not interact strongly. Members of the lepton family include the electron and its neutrino, the muon and its neutrino, and the antiparticles of each of the above particles.
• **Meson:** a family of intermediate mass particles which mediate the strong interaction between baryons.
• **Hadron:** any particle which may participate in the strong interaction, i.e., a baryon or a meson.
• **Muon:** a lepton which is identical to an electron, except that it is roughly 200 times more massive.
• **Neutrino:** a virtually massless lepton which comes in three varieties (an electron's neutrino, a muon's neutrino and a taon's neutrino).
• **Pair production:** the creation of a particle-antiparticle pair out of energy.
• **Pair annihilation:** the conversion of a particle-antiparticle pair into two or more photons.
• **Fundamental forces:** four basic physical interactions between elementary particles which include strong, electromagnetic, weak, and gravitational interactions.
• **Electromagnetic interaction:** an interaction between elementary particles mediated by the exchange of a photon. If you notice that a photon is involved in an reaction, then you can be assured that the reaction involves the electromagnetic interaction.
• **Strong interaction:** an interaction between elementary particles mediated by the exchange of a meson between two baryons or two mesons. Any reaction will go via the strong interaction unless at least one of the particles involved does not take part in the interaction.
• **Weak interaction:** a fundamental interaction between elementary particles that is weaker than the strong and electromagnetic interaction. This interaction is responsible for the radioactive decay of many of the elementary particles.
• **Gravitational Interaction:** The fourth and weakest fundamental force is the "gravitational interaction." All of the elementary particles, including the massless photon and the neutrino, take part in the gravitational interaction. However, on the elementary particles scale the gravitational interaction is negligible compared with the other forces.

Calculation in natural units have shown that the relative strengths (by a parameter α) of the four fundamental interactions are in the order

$$\alpha_{Strong} : \alpha_{EM} : \alpha_{Weak} : \alpha_{Grav} = 1 : 10^{-2} : 10^{-7} : 10^{-38}$$

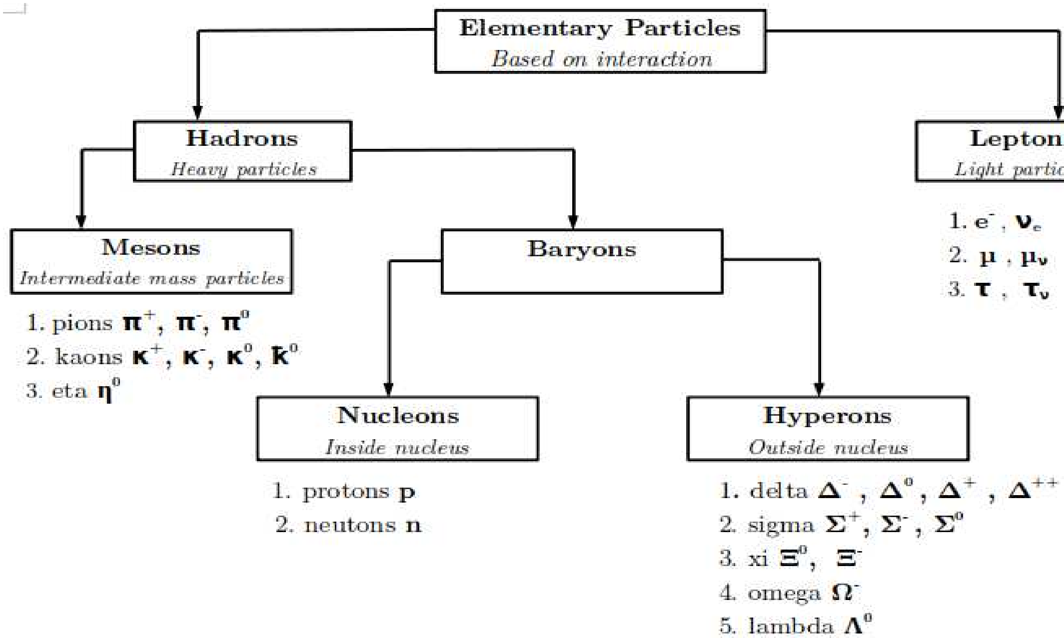
8.2 Classification of Elementary particle

Two types of statistics are used to describe elementary particles, and the particles are classified on the basis of which statistics they obey.

• **Fermi-Dirac statistics** apply to those particles restricted by the Pauli exclusion principle ; particles obeying the Fermi-Dirac statistics are known as fermions. Leptons and quarks are fermions. Two fermions are not allowed to occupy the same quantum state. In general, fermions compose nuclear and atomic structure.

• **Bose-Einstein statistics** apply to all particles not covered by the exclusion principle, and such particles are known as bosons. The number of bosons in a given quantum state is not restricted. In general, bosons act to transmit forces between fermions; the photon, gluon, the W, Z and Higgs particles are bosons.

Basic categories of particles have also been distinguished according to other particle behavior. The strongly interacting particles were classified as either mesons or baryons ; it is now known that mesons consist of quark-antiquark pairs and that baryons consist of quark triplets. The meson class members are more massive than the leptons but generally less massive than the proton and neutron, although some mesons are heavier than these particles. The lightest members of the baryon class are the proton and neutron, and the heavier members are known as hyperons. In the meson and baryon classes are included a number of particles that cannot be detected directly because their lifetimes are so short that they leave no tracks in a cloud chamber or bubble chamber . These particles are known as resonances, or resonance states, because of an analogy between their manner of creation and the resonance of an electrical circuit. The following figure shows all the particles



8.3 Conservation laws for Elementary particle

Conservation laws are critical to an understanding of particle physics. Strong evidence exists that energy, momentum, and angular momentum are all conserved in all particle interactions. The annihilation of an electron and positron at rest, for example, cannot produce just one photon because this violates the conservation of linear momentum. The special theory of relativity modifies definitions of momentum, energy, and other familiar quantities. In particular, the relativistic momentum of a particle differs from its classical momentum by a factor $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ that varies from 1 to ∞ , depending on the speed of the particle. But whatever is the quantity that is getting conserved is always leads to some symmetry. Closely related to conservation laws are three **symmetry** principles that apply to changing the total circumstances of an event rather than changing a particular quantity. The three symmetry operations associated with these principles are: **charge conjugation (C)**, which is equivalent to exchanging particles and antiparticles; **parity (P)**, which is a kind of mirror-image symmetry involving the exchange of left and right; and **time-reversal (T)**, which reverses the order in which events occur. According to the symmetry principles (or invariance principles), performing one of these symmetry operations on a possible particle reaction should result in a second reaction that is also possible. However, it was found in 1956 that parity is not conserved in the weak interactions, i.e., there are some possible particle decays whose mirror-image counterparts do not occur. Although not conserved individually, the combination of all three operations performed successively is conserved; this law is known as the **CPT theorem**.

8.3.1 Noether theorem

Noether theoram states that every symmetry in nature is related to a conservation law and *vice versa.* The following table gives the idea about Noether's theoram.

Invariance under	**Leads to**
Translations in time	conservation of energy
Translations in space	conservation of momentum.
Rotation in space	conservation of angular momentum
Gauge transformation	conservation of charge

8.3.2 Universally Conserved Quantities

Conservation of Momentum: If we consider the particles in a reaction and the forces between them as a closed, isolated system, then the system's total momentum must be conserved. If the reacting particle decays into two products, then the two resulting ("product") particles must move in opposite directions with momenta of equal magnitude, when observed in the rest frame of the decaying particle. If the initial particle decays into three products, then the momenta of the three product particles must add to zero and hence must be coplanar, when observed in the rest frame of the decaying particle.

Conservation of Energy: With the common assumption that the interacting system is closed and isolated, total energy is conserved in a particle reaction. Relativistic calculations are required to verify that the energy actually is conserved in any given reaction. However we may make these observations:
• For two or more colliding particles, the reaction is energetically possible if sufficient kinetic energy is supplied to the reaction.
• For particle decays, conservation of energy requires that the mass of the decay products be less than or at most no greater than the mass of the decaying particle. This can be seen by considering the decaying particle in its rest frame. Before the decay its total energy is merely its rest energy, mc^2. After the decay, the product particles typically have some kinetic energy. To balance energy on both sides of the reaction equation, the total rest energy of the products must be less than the rest energy of the decaying particle. This may be stated mathematically as: $Mc^2 = \sum\limits_{products} (mc^2 + E_k)$. Since $E_k \geq 0$, the total mass of the products must be less than the mass M of the initial particle.

Conservation of Angular Momentum: In the absence of any external torques, the total angular momentum of an isolated system of interacting particles must be conserved. To check for conservation of angular momentum, you need to know the spins of the particles involved and the rules for adding quantized angular momenta. Use the following facts obtained from the rules for adding quantized angular momenta:
• The total angular momentum of two particles of integer spin ($S = 0, 1, 2, \ldots$) is an integral multiple of $\hbar$.
• The total angular momentum of two particles of half-odd-integer spin ($S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$) is also an integral multiple of $\hbar$.
• The total angular momentum of two particles, one of integer spin and the other of half-integer spin, is a half-odd-integer multiple of $\hbar$.
As an example, consider the reaction $p + \pi^- \to e^- + e^+$. The spin of the proton is $\frac{1}{2}$ and the spin of the π is zero, so the total angular momentum of the reacting particles is a half-odd-integer multiple of $\hbar$. However the electron and positron each have spin $\frac{1}{2}$, so the resulting particles have a total angular momentum that is an integer multiple of $\hbar$. Angular momentum is not conserved, so this reaction will never take place.

Conservation of Charge: All elementary particle reactions must conserve charge. Unless you are considering the sub-hadronic world, all charged elementary particles have a charge that is a positive or negative integer multiple of the electron's charge. Given the charge of all particles involved in a reaction, the net charge of the initial particles must equal the net charge of the final particles.
Other conservation laws have meaning only on the level of particle physics, including the three conservation laws for leptons, which govern members of the electron, muon, and tau families respectively, and the law governing members of the baryon class. New quantities have been invented to explain certain aspects of particle behavior. For example, the relatively slow decay of kaons, lambda hyperons, and some other particles led physicists to the conclusion that some conservation law prevented these particles from decaying rapidly through the strong interaction; instead they decayed through the weak interaction. This new quantity was named **strangeness** and is conserved in both strong and electromagnetic interactions, but not in weak interactions. Thus, the decay of a strange particle into nonstrange particles, e.g., the lambda baryon into a proton and pion, can proceed only by the slow weak interaction and not by the strong interaction.

8.3.3 Conservation of Family particle number

Conservation of Baryon Number: A conservation law which accounts for the stability of the proton is the conservation of "**baryon number,**" **B**. This also accounts for the fact that the neutron and all of the other heavier "elementary particles," the baryons, decay in such a way that the final product is the proton. This conservation law is similar to electrical charge conservation. Just as all particles can be assigned electrical charge values of $0, \pm 1$, or ± 2, etc., (in units of the quantum of electric charge, the charge on the protons), every particle has a "baryon charge" of $B = 0$, $B = +1$, or $B = -1$. Furthermore, in any reaction the total baryon number of the products of the reaction must equal the sum of the baryon numbers of the initial particles. The proton is the lightest particle with baryon charge $B = +1$ so this accounts for the stability of the proton. All baryons have $B = 1$, their anti-particles have $B = -1$, and all mesons, leptons and the photon have $B = 0$. Thus

$$\begin{array}{rcccc} & p & \rightarrow \pi^+ & + & \pi^0 \\ \text{B:} & +1 & 0 & & 0 \end{array}$$

is forbidden by baryon conservation, while

$$\begin{array}{rcccc} & \Sigma^+ & \rightarrow p & + & \pi^0 \\ \text{B:} & +1 & +1 & & 0 \end{array} \qquad\qquad \begin{array}{rcccccc} & n & \rightarrow p & + & e^- & + & \bar{\nu}_e \\ \text{B:} & +1 & +1 & & 0 & & 0 \end{array}$$

are allowed by baryon number conservation as well as by all other conservation laws.

Conservation of Lepton Numbers: There appears to be a set of two quantum numbers associated with leptons that has similarities to baryon number. These two quantum numbers are similar to baryon number in the sense that the conservation laws associated with them are absolute; they must be satisfied in all processes. These quantum numbers are the electron lepton number and the muon lepton number. Their assigned values are:

Particle	**e^- Number**	**μ Number**	**τ Number**
e^-	+1	0	0
μ^-	0	+1	0
τ^-	0	0	+1
ν_e	+1	0	0
ν_μ	0	+1	0
ν_τ	0	0	+1

The antiparticle to any of these particles has the opposite lepton number; for example, the e^+ , the anti-electron, has an electron lepton number of -1. All other particles have zero value for both these lepton numbers.

Conservation of Strangeness: In 1947 the British physicists Rochester and Butler observed new particles in cosmic ray events. These particles came in two forms: a neutral one that decayed into a π^+ and a π^-, and a positively charge one that decayed into a μ^+ and a photon. Laboratory equipments are not sufficient to produce these and they are also very short-lived. Then how come these gets produced in cosmic sources? So kind of a strange behaviour! So particle physicist had assigned a kind hadron conservation number called **strangeness, *S*** which are associated with these strange particles because they have found that certain hadrons to decay via the strong interaction can't be only described with the exsisting conservation rules. Analogous to the assignment of baryon number, the strangeness assignments are to be made in a way which is consistent with what is observed. That is, those processes involving hadrons which are observed not to go via the strong interaction must violate Strangeness conservation. Thus this conservation law is absolute only for Strong Interaction and Electromagnetic processes; it may be violated in processes which go via the weak interaction.

Conservation of Isospin: The assignment of isospin quantum numbers, I may be made if we consider the situation which prevails with the spectrum of hadrons (baryons and mesons). The assignment of isospin quantum numbers would be an empty exercise except for the observation that I is conserved in all strong interactions and third component of isospin, I_3 is conserved in all strong and electromagnetic interactions.

8.3.4 Gell-mann-Nishijima Formula

The hadrons are divided into two broad categories called mesons (integer spin) and baryons (half-odd integral spin with an additional quantum number called the baryon number). We have already alluded to the isospin and strangeness before. As far as strong interactions are concerned both isospin and strangeness are conserved exactly. By inspection, it is easy to see that there exists a relation between the charges of the particles and other quantum numbers. The Gell-Mann-Nishijima formula relates the baryon number B, the strangeness S, the isospin I_3 of quarks and hadrons to the electric charge Q. The original form of the Gell-Mann-Nishijima formula is:

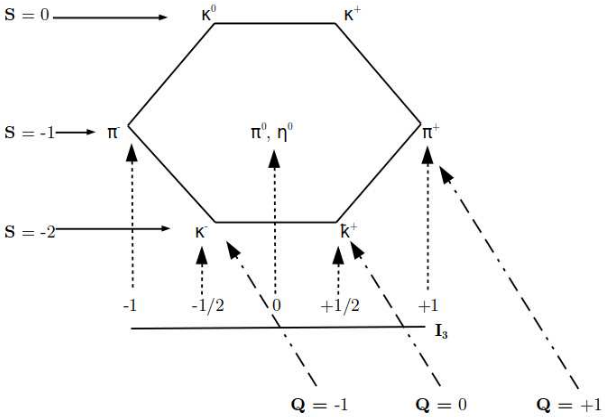
$$Q = I_3 + \frac{B + S}{2} \qquad \text{where} \quad (B + S) \text{ is called as } \textbf{hypercharge}$$

Note that Q, I_3, B, and S are all additive quantum numbers, and Gell-MannNishijima relation is a linear equation. Therefore, if the constituents which made up a particle satisfy this relation, then the bound states also satisfy the same relation. Nothing mysterious.

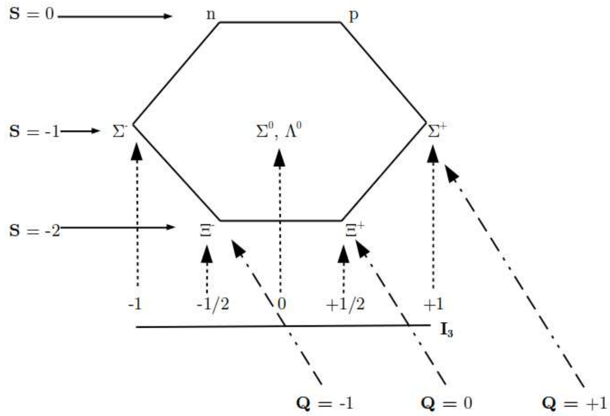
8.3.5 The Eightfold way

Many attempts have been made in trying to discover a system of organization that grouped elementary particles into larger groups of identification. The only one that has had much success is the Eightfold Way which is adopted from Buddhism. It was proposed independently in 1961 by both Gell-Mann and Yuval Ne'eman, the Eightfold Way groups the baryons and mesons into geometrical patterns of the same baryon number, spin and parity. An example of one of these shapes is the baryon octet which consists of the eight lightest baryons.

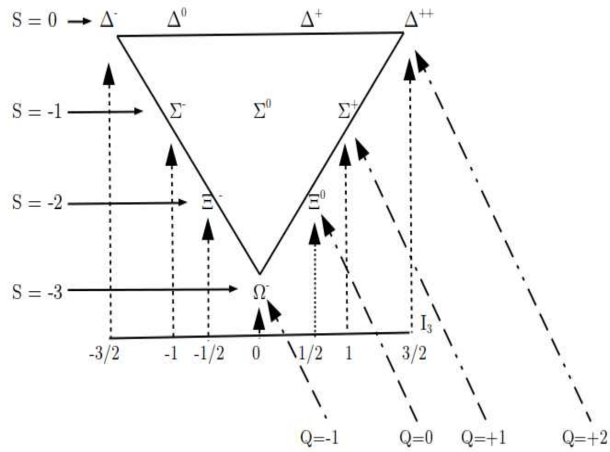
• **Meson Octet:** The eightfold way organizes eight of the lowest spin-0 mesons into an octet. Diametrically opposite particles in the diagram are anti-particles of one-another while particles in the center are their own anti-particle.



• **Baryon Octet:** The eightfold way organizes the spin-$\frac{1}{2}$ baryons into an octet. Diametrically opposite particles in the diagram are anti-particles of one-another while particles in the center are their own anti-particle.



• **Baryon decuplet:** The principles of the eightfold way also applied to the spin-$\frac{3}{2}$ baryons, forming a decuplet. However, one of the particles of this decuplet had never been previously observed when the eightfold way was proposed. Gell-Mann called this particle the Ω^- and predicted in 1962 that it would have a strangeness -3, electric charge -1 and a mass near 1680 MeV/c^2. In 1964, a particle closely matching these predictions was discovered by a particle accelerator group at Brookhaven. Gell-Mann received the 1969 Nobel Prize in Physics for his work on the theory of elementary particles.



Thus the spectra of hadrons seem to show some pattern of $SU(3)$ symmetry. But this symmetry is lots worse than isospin symmetry of $SU(2)$ because the mass splitting within the $SU(3)$ multiplets is about 20% at best. Nevertheless, it is still useful to classify hadrons in terms of $SU(3)$ symmetry.

8.4 Quark Model

The hadrons are even made up of even by small structure which has been a possibility due to the following evidences

• The magnetic moments of proton and neutron are not $\mu_N = \dfrac{e\,\hbar}{2m_p}$ and 0 respectively which means that they are **not point-like**

• Electron-proton scattering at high p^2 deviates from Rutherford scattering indicates **proton has substructure**.

• Hadron jets are observed in laboratory during $e^+\,e^-$ and pp collisions. And

• One peculiar feature of the eight fold way is that octet and decuplet are not the fundamental representation of $SU(3)$ group.

Thus the model called quark model is a classification scheme for hadrons in terms of their valence quarks - the quarks and antiquarks which give rise to the quantum numbers of the hadrons which probably will verify all the above observations. The quark model in its modern form was developed by Murray Gell-Mann and independently by

proposed by George Zweig, in which all hadrons are built out of spin-$\frac{1}{2}$ quarks which transform as members of the fundamental representation of $SU(3)$.
The assumption of the model is that:
• Hadrons are not 'fundamental', but they are built from 'valence quarks', i.e. quarks and antiquarks, which give the quantum numbers of the hadrons.

$$|\text{Baryons}\rangle = |q\,q\,q\,\rangle \qquad \text{and} \qquad |\text{Mesons}\rangle = |q\,\bar{q}\,\rangle$$

That is baryons are made up of three quarks and mesons are made up of a quark and an antiquark. The following table explains the properties of six quark flavors.

Property	d	u	s	c	b	t
Q-Electric charge	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
I- isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
$\mathbf{I}_3$-isospin 3^{rd} component	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
S-strangeness	0	0	-1	0	0	0
C-charm	0	0	0	$+1$	0	0
$\mathcal{B}$-bottomness	0	0	0	0	-1	0
T-topness	0	0	0	0	0	$+1$

With the advent of these the Gell-mann-Nishijima formula takes the shape as

$$Q = I_3 + \frac{(B + S + C + \mathcal{B} + T)}{2} \qquad \text{where the bracketed term is called as } \textbf{hypercharge}$$

As for example under this scheme, mesons are $q\,\bar{q}$ bound states. So the quark composition of the meson octet is given below. I will also make an analysis of the scheme.

$$\pi^+ \equiv u\,\bar{d} \qquad \pi^- \equiv \bar{u}\,d \qquad \pi^0 \equiv \frac{1}{\sqrt{2}}(u\,\bar{u} - d\,\bar{d})$$

$$\kappa^+ \equiv u\,\bar{s} \qquad \kappa^- \equiv \bar{u}\,s \qquad \kappa^0 \equiv \bar{s}\,d \qquad \eta^0 \equiv \frac{1}{\sqrt{6}}(u\,\bar{u} + d\,\bar{d} - 2\,s\,\bar{s}) \qquad \bar{\kappa}^0 \equiv s\,\bar{s}$$

and baryons are $q\,q\,q$ bound states. The quark composition of the baryon octet is given below.

$$p \equiv u\,u\,d \qquad n \equiv d\,d\,u$$

$$\Sigma^+ \equiv s\,u\,u \qquad \Sigma^- \equiv s\,d\,d \qquad \Sigma^0 \equiv \frac{1}{\sqrt{2}}\,s\,(u\,d + d\,u)$$

$$\Xi^0 \equiv s\,s\,u \qquad \Xi^- \equiv s\,s\,d$$

$$\Lambda^0 \equiv \frac{1}{\sqrt{2}}\,s\,(u\,d - d\,u)$$

It seems that the quantum numbers of the hadrons are all carried by the quarks. But we do not know the dynamics which bound the quarks into hadrons. Since quarks are the fundamental constituent of hadrons it is important to find these particles. But over the years none have been found.

• **Paradoxes of quark model:**
• Quarks have fractional charges while all observed particles have integer charges. At least one of the quarks is stable. None has been found.
• Hadrons are exclusively made out $q\,\bar{q}$; $q\,q\,q$ bound states. In other word, $q\,q$; $q\,q\,q\,q$ states are absent.
• The quark content of the baryon $X^{\star\,++}$ is $u\,u\,u$. If we choose the spin states of these three, this will leads to violation of Pauli exclusion principle.

8.4.1 Gell-Mann Okubo mass formula

Since $SU(3)$ is not an exact symmetry, we want to see whether we can understand the pattern of the $SU(3)$ breaking. Experimentally, $SU(2)$ seems to be a good symmetry, we will assume isospin symmetry to set $m_u = m_d$: We will assume that we can write the hadron masses as linear combinations of quark masses.

• **For spin-0, odd parity mesons,** (0^- *meson*)**:**
Here we assume that the meson masses are linear functions of quark masses

$$m_\pi^2 = \lambda\,(m_o + 2\,m_u)$$
$$m_\kappa^2 = \lambda\,(m_o + m_u + m_s)$$
$$m_\eta^2 = \lambda\,(m_o + \frac{2}{3}\,(m_u + 2\,m_s)$$

where and m_0 and λ are some constants with mass dimension. Eliminate the quark masses we get

$$4\,m_\kappa^2 \,=\, m_u^2 \,+\, 3\,m_\eta^2$$

Experimentally, we have $4\,m_\kappa^2 \,\approx\, 0.98\,(GeV)^2$ while $m_u^2 \,+\, 3\,m_\eta^2 \,\approx\, 0.92\,(GeV)^2$ This seems to show that this formula works quite well.

• **For spin-$\frac{1}{2}$, even parity baryons, ($\frac{1}{2}^+\,baryon$):**
Here we assume that the meson masses are linear functions of quark masses

$$\begin{aligned} m_N \,&=\, (m_o \,+\, 3\,m_u) \\ m_\Sigma \,=\, m_\Lambda \,&=\, (m_o \,+\, 2\,m_u \,+\, m_s) \\ m_\Xi \,&=\, (m_o \,+\, 2\,m_u \,+\, 2\,m_s) \end{aligned}$$

where and m_0 is a constants with mass dimension. Eliminate the quark masses we get the Gell-Mann Okubo mass formula for baryons with spin-$\frac{1}{2}$ as

$$\frac{m_\Sigma \,+\, 3\,m_\Lambda}{2} \,=\, m_N \,+\, m_\Xi$$

Expermentally, $\dfrac{m_\Sigma \,+\, 3\,m_\Lambda}{2} \,\approx\, 2.23\,GeV$ and $m_N \,+\, m_\Xi \,\approx\, 2.25\,GeV$

• **For spin-$\frac{3}{2}$, even parity baryons, ($\frac{3}{2}^+\,baryon$):**
The mass relation here is quite simple. This sometimes is referred to as **equal spacing rule**. In fact when this relation is derived the particle Ω has not yet been found and this relation is used to predicted the mass of Ω and subsequent discovery gives a very strong support to the idea of $SU(3)$ symmetry. the Gell-Mann Okubo mass formula for baryons with spin-$\frac{3}{2}$ as

$$m_\Omega \,-\, m_\Xi \,=\, m_\Xi \,-\, m_\Sigma \,=\, m_\Sigma \,-\, m_N$$